parent ROADMAP

SUPPORTING YOUR CHILD IN GRADE SIX

MATHEMATICS
The way we taught students in the past simply does not prepare them for the higher demands of college and careers today and in the future. Your school and schools throughout the country are working to improve teaching and learning to ensure that all children will graduate high school with the skills they need to be successful.

In mathematics, this means three major changes. Teachers will concentrate on teaching a more focused set of major math concepts and skills. This will allow students time to master important ideas and skills in a more organized way throughout the year and from one grade to the next. It will also call for teachers to use rich and challenging math content and to engage students in solving real-world problems in order to inspire greater interest in mathematics.
In grade six, your child will learn the concept of rates and ratios and use these tools to solve word problems. Students will work on quickly and accurately dividing multi-digit whole numbers and adding, subtracting, multiplying, and dividing multi-digit decimals. Students will extend their previous work with fractions and decimals to understand the concept of rational numbers—any number that can be made by dividing one integer by another, such as $\frac{1}{2}$, 0.75, or 2. Students will also learn how to write and solve equations—mathematical statements using symbols, such as $20 + x = 35$—and apply these skills in solving multi-step word problems. Activities in these areas will include:

- Understanding and applying the concepts of ratios and unit rates, and using the correct language to describe them (for example, the ratio of wings to beaks in a flock of birds is 2 to 1, because for every 2 wings there is 1 beak)
- Building on knowledge of multiplication and division to divide fractions by fractions
- Understanding that positive and negative numbers are located on opposite sides of 0 on a number line
- Using pairs of numbers, including negative numbers, as coordinates for locating or placing a point on a graph
- Writing and determining the value of expressions with whole-number exponents (such as $15 + 3^2$)
- Identifying and writing equivalent mathematical expressions by applying the properties of operations. For example, recognizing that $2(3 + x)$ is the same as $6 + 2x$
- Understanding that solving an equation such as $2 + x = 12$ means answering the question, “What number does $x$ have to be to make this statement true?”
- Representing and analyzing the relationships between independent and dependent variables
- Solving problems involving area and volume

A dependent variable is a number whose value depends on other factors, while the value of an independent variable is set. For example, in a problem involving a constant speed (such as 60 mph), students may be asked how many miles will be travelled in 30 minutes. Since distance is determined by time, distance is the dependent variable, and time is the independent variable.

Don’t be afraid to reach out to your child’s teacher—you are an important part of your child’s education. Ask to see a sample of your child’s work or bring a sample with you. Ask the teacher questions like:

- Where is my child excelling? How can I support this success?
- What do you think is giving my child the most trouble? How can I help my child improve in this area?
- What can I do to help my child with upcoming work?
Here are just a few examples of how students will learn about and work with fractions in grade six.

<table>
<thead>
<tr>
<th>Grade Five Mathematics</th>
<th>Grade Six Mathematics</th>
<th>Grade Seven Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Add and subtract fractions with different denominators (bottom numbers)</td>
<td>• Divide fractions by fractions using models and equations to represent the problem</td>
<td>• Add, subtract, multiply, and divide rational numbers in any form, including whole numbers, fractions, and decimals)</td>
</tr>
<tr>
<td>• Multiply a fraction by a whole number or another fraction</td>
<td>• Solve word problems involving division of fractions by fractions</td>
<td>• Solve multi-step problems involving positive and negative rational numbers</td>
</tr>
<tr>
<td>• Divide fractions by whole numbers and whole numbers by fractions to solve word problems</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Real-world problems give students a context for dividing fractions by fractions.

**Example of a problem involving the division of fractions.**

Ann has 3 ½ lbs of peanuts for the party. She wants to put them in small bags each containing ½ lb. How many small bags of peanuts will she have?

Students use their knowledge of fractions to see that there are 7 halves in 3 ½ lbs, so there will be 7 bags of peanuts.

Students can also find how many halves are in 3 ½ by applying the traditional procedure of dividing 3 ½ by ½.

\[
3 \frac{1}{2} = \frac{7}{2} \\
\frac{7}{2} \div \frac{1}{2} = \frac{7}{2} \times \frac{2}{1} = \frac{14}{2} = 7
\]
Here are just a few examples of how students will develop an understanding of ratios and proportions in grade six.

<table>
<thead>
<tr>
<th>Grade Five Mathematics</th>
<th>Grade Six Mathematics</th>
<th>Grade Seven Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Explain why a fraction is equal to another fraction</td>
<td>• Understand the concept of a ratio and use the correct language to describe it</td>
<td>• Analyze proportional relationships and use them to solve real-world problems</td>
</tr>
<tr>
<td>• Interpret multiplication as scaling (resizing)</td>
<td>• Understand the concept of a unit rate (the rate per unit, or a ratio with a denominator of 1) and use the correct language to describe it</td>
<td>• Calculate the unit rates associated with ratios of fractions, such as the ratio of ( \frac{3}{2} ) a mile for every ( \frac{1}{4} ) of an hour</td>
</tr>
<tr>
<td></td>
<td>• Use ratio and rates to solve real-world problems</td>
<td>• Recognize and represent proportional relationships in various ways, including using tables, graphs, and equations</td>
</tr>
</tbody>
</table>

Students use diagrams and tables to think through and solve real-world problems involving ratios.

**Example of a problem involving ratios**

A slime mixture is made by mixing glue and liquid laundry starch in a ratio of 3 to 2. How much glue and how much starch are needed to make 90 cups of slime?

<table>
<thead>
<tr>
<th>Glue</th>
<th>Starch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parts</td>
<td>Quantities</td>
</tr>
<tr>
<td>5 parts</td>
<td>90 cups</td>
</tr>
<tr>
<td>1 part</td>
<td>( \frac{90}{5} = 18 ) cups</td>
</tr>
<tr>
<td>2 parts</td>
<td>( 2 \times 18 = 36 ) cups</td>
</tr>
<tr>
<td>3 parts</td>
<td>( 3 \times 18 = 54 ) cups</td>
</tr>
</tbody>
</table>

Using knowledge of ratios and proportions, students see that if each cup of slime is made up of 3 parts glue and 2 parts starch, there are 5 parts in each cup. They can then compute the quantity of one, two, and three parts of 90 cups to determine the exact amounts of glue and starch needed.
Helping your child learn outside of school

1. Ask your child to calculate the unit rates of items purchased from the grocery store. For example, if 2 pounds of flour cost $3.00, how much does flour cost per pound?

2. Have your child determine the amount of ingredients needed when cooking. For example, if a recipe calls for 8 cups of rice to serve 4 people, how many cups of rice do you need to serve 6 people?

3. Encourage your child to stick with it whenever a problem seems difficult. This will help your child see that everyone can learn math.

4. Praise your child when he or she makes an effort, and share in the excitement when he or she solves a problem or understands something for the first time.

Additional Resources

For more information on the Common Core State Standards for mathematics, go to http://www.corestandards.org/Math/ or http://www.commoncoreworks.org.

For more information on the standards in mathematics related to ratios/proportions or fractions, go to http://commoncoretools.me/category/progressions/.

For math games and challenges to do at home, go to http://www.figurethis.org/download.htm or www.24game.com.
Welcome to Grade 6 and *A Story of Ratios™*! In the first topic of Module 1, students will become familiar with ratio language (how to describe ratios) and notation (how to write ratios, e.g., 3:2 or 7 to 15). They will learn how to identify equivalent ratios (e.g., 1:2, 2:4, and 4:8) and to solve problems with ratio values.

You can expect to see homework that asks your child to do the following:

- Write ratios using correct ratio notation.
- Describe the ratio relationship.
- Write and identify equivalent ratios. (Students may use a tape diagram.)
- Define and determine the value of the ratio to determine whether ratios are equivalent.
- Solve word problems involving ratios.

**SAMPLE PROBLEM** *(From Lesson 5)*

In the month of August, a total of 192 registrations were purchased for passenger cars and pickup trucks at the local Department of Motor Vehicles (DMV). The DMV reported that in the month of August, for every 5 passenger cars registered, there were 7 pickup trucks registered. How many of each type of vehicle were registered in the county in the month of August?

1. Using the information in the problem, write any two ratios and describe the meaning of each.

   *The ratio of cars to trucks is 5:7 and is a part-to-part ratio. The ratio of trucks to total vehicles is 7 to 12 and is a part-to-whole ratio.*

2. Make a tape diagram that represents the quantities in the part-to-part ratio that you wrote.

   - **Passenger Cars**
     - ![Tape Diagram](image)
   - **Pickup Trucks**
     - ![Tape Diagram](image)

3. What value does each individual part of the tape diagram represent?

   *Divide the total quantity into 12 equal-sized parts:*
   
   \[
   \frac{192}{12} = 16
   \]

4. How many of each type of vehicle were registered in August?

   - \(5 \times 16 = 80\) *There were 80 passenger cars registered in August.*
   - \(7 \times 16 = 112\) *There were 112 pickup trucks registered in August.*

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.
HOW YOU CAN HELP AT HOME

- Determine and write the ratio for the number of cups of flour to the number of cups of sugar in a recipe that calls for 3 cups of flour and 1 cup of sugar. Use the ratio to determine the number of cups of flour needed if someone makes multiple batches using 6 cups of sugar (18). Ask your child to explain how to use a tape diagram to solve this problem.

- Have your child make ratio cards by writing ratios (e.g., 2:5) on notecards or flashcards (one ratio per card). Show the cards one at a time, and ask your child to generate three equivalent ratios for each card (e.g., 4:10, 14:35, 20:50). For an added challenge, ask your child what the nonzero number \( c \) is for each set of equivalent ratios (2, 7, 10).

TERMS

Equivalent ratios: Ratios that have the same value; for example, 1:3, 2:6, and 3:9 are equivalent ratios.

Multiplicative comparisons: Comparisons that describe the relationship between two quantities in terms of multiples; for example, “twice as many apples as oranges” or “three times as many cats as chipmunks” are multiplicative comparisons.

Nonzero number \( c \): The number that is multiplied by each part of the ratio to make an equivalent ratio.

Quantities: Amounts, or measurements, such as length, area, volume, and speed.

Quotient: The answer to a division problem.

Ratio: A statement of how two (nonzero) numbers compare. They can be written as \( A:B \) or \( A \) to \( B \).

Ratio relationship: The relationship between two quantities in a given setting; for example, sugar to butter in a recipe or paws to tails in the monkey house at the zoo. It is also the set of all ratios that are the same (equivalent). A ratio of 1:4, for example, can be used to describe ratio relationships (1:4, 2:8, 3:12) and can be represented by various models (ratio tables, double number line diagrams, and by equations and their graphs) as shown in the Models section.

Value of the ratio: For the ratio \( A:B \), the value of the ratio is the quotient \( \frac{A}{B} \) where \( B \neq 0 \). For example, the ratio 6:10 has a value of \( \frac{6}{10} \) or 0.6.

MODELS

Tape Diagram

2

2:3, 4:6, 6:9

Cups of Sugar

Cups of Flour

3
KEY CONCEPT OVERVIEW

In Topic B, students continue their work with equivalent ratios, representing equivalent ratios in various ways, and noting the advantages and disadvantages of each model: ratio tables, tape diagrams, double number line diagrams, equations, and graphs on coordinate planes. Students also study the additive and multiplicative patterns of the ratios presented in ratio tables.

You can expect to see homework that asks your child to do the following:

- Make a ratio table presenting equivalent ratios.
- Use a ratio table to answer questions and make comparisons.
- Use various models to answer questions.
- Write an equation to represent a situation.
- Locate the value of the ratio in a table or an equation.
- Make a graph from the ratio table.

SAMPLE PROBLEM (From Lesson 15)

Also on the news broadcast, a chef from a local Italian restaurant demonstrated how he makes fresh pasta daily for his restaurant. The recipe for his pasta is below.

3 eggs, beaten
1 teaspoon salt
2 cups all-purpose flour
2 tablespoons water
2 tablespoons vegetable oil

Determine the ratio of the number of tablespoons of water to the number of eggs. 2:3

Provided the information in the table below, complete the table to determine ordered pairs. Use the ordered pairs to graph the relationship of the number of tablespoons of water to the number of eggs.

<table>
<thead>
<tr>
<th>Tablespoons of Water</th>
<th>Number of Eggs</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>12</td>
<td>18</td>
</tr>
</tbody>
</table>

Additional sample problems with detailed answer steps are found in the Eureka Math Homework Helpers books. Learn more at GreatMinds.org.

For more resources, visit » Eureka.support
HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are just a few tips to help you get started:

- Ask your child to explain the advantage(s) and disadvantage(s) of using a table, double number line diagram, equation, or graph to represent equivalent ratios. For example, a table presents the information in an organized way but doesn’t necessarily show the relationship between the two values as nicely as a graph does.

- Have your child draw a ratio table (with 5 rows) and complete it using five equivalent ratios. Provide or ask your child to generate a context; for example, you might state that a lemonade recipe calls for one cup of lemon juice for every six cups of water and have your child create a table showing 1:6 and four equivalent ratios. Next, ask your child to graph the ordered pairs on a coordinate plane (using graph paper) and find the value of the ratio. Finally, challenge your child to write an equation using the value of the ratio, and explain her work to you.

- Team up with your child. Make ratio cards by writing ratios on notecards or flashcards. Hold up the cards one at a time. State an equivalent ratio for each card, and challenge your child to state a different equivalent ratio. Ask your child to identify the nonzero number \( c \) for each set of equivalent ratios and to explain how he figured this out.

TERMS

Additive Comparison: Description of the relationships between two quantities or amounts by asking or telling how much/many more (or less) one is compared to the other (e.g., “three more apples than oranges”).

MODELS

Coordinate Plane

![Coordinate Plane Diagram]

Double Number Line Diagram

![Double Number Line Diagram]

Equation

\[
R = 4B \\
B = \frac{1}{4}R
\]

Multiplicative Structure of the Ratio Table

<table>
<thead>
<tr>
<th>Gallons of Red Paint</th>
<th>Gallons of White Paint</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 ( \times 4 )</td>
<td>12</td>
</tr>
<tr>
<td>6 ( \times 4 )</td>
<td>24</td>
</tr>
<tr>
<td>12 ( \times 4 )</td>
<td>48</td>
</tr>
<tr>
<td>21 ( \times 4 )</td>
<td>84</td>
</tr>
</tbody>
</table>

Ratio Table

<table>
<thead>
<tr>
<th>Cups of Sugar</th>
<th>Cups of Flour</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>
**KEY CONCEPT OVERVIEW**

In Topic C, students will extend their knowledge of rate as they focus on **unit rate**. They will solve real-world word problems involving unit pricing, constant speed, and constant rates of work. Students will also learn to convert units of measurement (ounces to pounds or feet to inches) in order to make comparisons. Last, they will use their understanding of unit rates and conversions to interpret and model real-world scenarios.

You can expect to see homework that asks your child to do the following:

- Determine the unit rate and use it to answer questions, using models from previous topics.
- Convert measurement units using rates.
- Compare rates using tables, equations, and graphs.
- Locate the unit rate using tables, equations, and graphs.
- Create a graph using the unit rate and equivalent ratios.
- Use the equation \( d = rt \) (distance = rate × time) to solve problems.

**SAMPLE PROBLEM** *(From Lesson 20)*

Emilia and Miranda are sisters, and their mother just signed them up for a new cell phone plan because they send too many text messages. Using the information below, determine which sister sends the most text messages. How many more text messages does this sister send per week?

**Emilia:**

<table>
<thead>
<tr>
<th>Number of Weeks</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Text Messages</td>
<td>1,200</td>
<td>2,400</td>
<td>3,600</td>
<td>4,800</td>
</tr>
</tbody>
</table>

**Miranda:** \( m = 410w \), where \( w \) represents the number of weeks, and \( m \) represents the number of text messages.

**Emilia:**

\[
\frac{2400 - 1200}{6 - 3} = \frac{1200}{3} = 400
\]

**Miranda sends 400 text messages per week.**

**Miranda sends 10 more text messages per week than Emilia.**

Additional sample problems with detailed answer steps are found in the Eureka Math Homework Helpers books. Learn more at GreatMinds.org.
HOW YOU CAN HELP AT HOME

You can help your child at home in many ways. Here are just a few tips to help you get started:

■ With the help of your vehicle’s odometer (which measures distance) and a clock, challenge your child to calculate the rate (distance divided by time) of a trip. For example, if you traveled 6 miles in 15 minutes, you traveled at a rate of 0.4 mile per minute (6 miles ÷ 15 minutes). Then, ask your child to calculate the rate of another trip and compare the two rates. Are the rates the same? If not, discuss reasons that the rates are different from one another.

■ Write a journal entry, draw a comic strip, or write song lyrics with your child to explain where the unit rate can be located in tables, graphs, and equations.

■ As a family, set a timer and do as many jumping jacks as you can in two minutes. Ask each person to keep track of how many jumping jacks are completed during that time. With your child, calculate each family member’s rate. Instead of jumping jacks, try hopping on one or both feet, doing squats, or any other activity you can time. (It doesn’t need to be a physical activity!) For an added math challenge, consider assigning each family member a different amount of time. Be creative and have fun!

TERMS

**Unit rate**: The numerical part of a rate measurement; for example, in the rate 45 mph, the unit rate is 45.
KEY CONCEPT OVERVIEW

In Topic D, students begin to explore a concept we know and use often in daily life—percents. Students are introduced to percents, learning how to find the percent of a quantity as the rate per 100, express a fraction as a percent, and connect percents to ratios. They also find percents of quantities in real-world contexts.

You can expect to see homework that asks your child to do the following:

- Write a percent as a fraction, decimal, or ratio.
- Use a model to answer problems about percents.
- Find the percent of a quantity.

SAMPLE PROBLEMS (From Lessons 24 and 25)

Robb’s Fruit Farm consists of 100 acres on which three different types of apples grow. On 25 acres, the farm grows Empire apples. McIntosh apples grow on 30% of the farm. Fuji apples are grown on the remainder of the farm. Shade in the grid to the left to represent the portion of the farm each apple type occupies. Use a different color for each type of apple. Create a key to identify which color represents each type of apple.

<table>
<thead>
<tr>
<th>B</th>
<th>B</th>
<th>G</th>
<th>G</th>
<th>G</th>
<th>G</th>
<th>G</th>
<th>P</th>
<th>P</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
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<td>G</td>
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<tr>
<td>B</td>
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<td>P</td>
<td>P</td>
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<tr>
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<td>B</td>
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<td>P</td>
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<td>B</td>
<td>B</td>
<td>B</td>
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<td>G</td>
<td>G</td>
<td>P</td>
<td>P</td>
<td>P</td>
</tr>
</tbody>
</table>

Robb’s Fruit Farm consists of 100 acres on which three different types of apples grow. On 25 acres, the farm grows Empire apples. McIntosh apples grow on 30% of the farm. Fuji apples are grown on the remainder of the farm. Shade in the grid to the left to represent the portion of the farm each apple type occupies. Use a different color for each type of apple. Create a key to identify which color represents each type of apple.

<table>
<thead>
<tr>
<th>Color Key</th>
<th>Part-to-Whole Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empire</td>
<td>Black (B) 25:100</td>
</tr>
<tr>
<td>McIntosh</td>
<td>Purple (P) 30:100</td>
</tr>
<tr>
<td>Fuji</td>
<td>Green (G) 45:100</td>
</tr>
</tbody>
</table>

For more resources, visit » Eureka.support
A company distributed a survey that asked participants whether or not they were happy with their jobs. Three hundred participants were unhappy with their jobs, while 700 participants were happy. Give a part-to-whole fraction for comparing happy participants to the whole. Then write a part-to-whole fraction comparing unhappy participants to the whole. What percent of the group were happy with their jobs? What percent were unhappy?

<table>
<thead>
<tr>
<th>Happy</th>
<th>Unhappy</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{700}{1,000} )</td>
<td>( \frac{300}{1,000} )</td>
</tr>
<tr>
<td>70%</td>
<td>30%</td>
</tr>
</tbody>
</table>

Create a model to justify your answer.

Additional sample problems with detailed answer steps are found in the Eureka Math Homework Helpers books. Learn more at GreatMinds.org.

**HOW YOU CAN HELP AT HOME**

You can help at home in many ways. Here are just a few tips to help you get started:

- Ask your child to share two ways each that she can write 5%, 40%, 72%, and 89% (or any other percentages). She may choose to write the percentages as fractions, decimals, or ratios. Encourage her to provide at least one example of each way. If necessary, remind her of the ways the class has worked with percentages throughout the topic (models, fractions, decimals, ratios).

- You and your child can each use a dry erase board to write a different fraction, decimal, or percent. Then, share with one another and determine which value is closer to 0, \( \frac{1}{2} \), or 1.

- Find ways percentages are used in your daily life. For example, while dining at a restaurant, challenge your child to calculate 10%, 15%, or 20% of the bill so you can determine the tip for the waiter/waitress. If the bill lists suggested tips, discuss how these values were calculated.

**TERMS**

**Percent:** One part in every hundred. One out of 100 is written as \( \frac{1}{100} \) and 1%. Percentages can be used as rates. For example, 30% of a quantity means \( \frac{30}{100} \) times the quantity.
KEY CONCEPT OVERVIEW

In the first topic of Module 2, students work extensively with division of fractions and mixed numbers. They create division stories, solve word problems, and study patterns to explore the relationship between multiplication and division, using familiar models such as tape diagrams, arrays, and number line diagrams.

You can expect to see homework that asks your child to do the following:

- Divide a fraction by a whole number.
- Solve word problems involving division of fractions.
- Use models to help solve problems.
- Rewrite a division expression (e.g., $\frac{9}{12} \div \frac{3}{12}$) in unit language (9 twelfths ÷ 3 twelfths).
- Write a partitive or measurement division story problem, for example, “Twenty-four students formed six equal-sized teams. How many students were on each team?”
- Use the standard algorithm for dividing fractions—invert the divisor (the second fraction) and multiply it by the first fraction (e.g., $\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$).
- Calculate the quotient.

SAMPLE PROBLEM  (From Lesson 2)

A construction company is setting up signs on two miles of a road. If the company places a sign at every $\frac{1}{4}$ mile, how many signs will it use?

$$2 \div \frac{1}{4} \quad \text{How many one-fourths in 2?}$$

$$2 \div \frac{1}{4} = 8 \text{ fourths} \div 1 \text{ fourth} = 8$$

*There are 8 fourths in 2. The company will use eight signs.*

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For more resources, visit » Eureka.support
HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here is a tip to help you get started.

- Write a story problem with your child that illustrates division. For example, for \( \frac{1}{2} \div \frac{1}{8} \), your story might be, “Bailey has a total of \( \frac{1}{2} \) pound of chocolate. She needs \( \frac{1}{8} \) pound of chocolate for each batch of brownies she bakes. How many batches of brownies can Bailey bake with \( \frac{1}{2} \) pound of chocolate?” Which language arts skills and strategies can your child incorporate into the story problem? Be creative and have fun!

TERMS

**Dividend**: The number that is divided by another number. For example, in the expression \( 32 \div 4 \), the number 32 is the dividend.

**Divisor**: The number by which another number is divided. In the problem \( 36 \div 9 = 4 \), 9 is the divisor.

**Measurement division**: Finding the number of groups when the number of items per group is known. For example, “How many one-fifths are in 7 wholes?”

**Multiplicative inverse**: When multiplying a number by its multiplicative inverse, the product (answer) is one. For example, \( \frac{3}{4} \) and \( \frac{4}{3} \) are multiplicative inverses because \( \frac{3}{4} \times \frac{4}{3} = 1 \).

**Partitive division**: Finding the number of items in each group when the number of groups is known. For example, “There are 12 apples divided evenly among three bags. How many apples are in each bag?”

**Quotient**: The answer to a division problem.

**Unit form**: Place value counting. For example, 34 can be stated as 3 tens 4 ones.

**Unit language**: Using the unit (e.g., thirds, fifths, tenths) to describe a number. For example, 0.4 is 4 tenths and \( \frac{1}{5} \) is 1 fifth.

MODELS

**Array Model**

![Array Model Image]

**Fraction Tiles**

![Fraction Tiles Image]

**Tape Diagram**

![Tape Diagram Image]

**Number Line Diagram**

![Number Line Diagram Image]
TIPS FOR PARENTS

KEY CONCEPT OVERVIEW

In Topic B, students revisit decimal operations (addition, subtraction, and multiplication) to prepare for decimal division. Students also encounter problems involving mixed numbers. They discover that when they convert mixed numbers to decimals, they can solve problems by using the standard algorithm for addition and subtraction of decimals and lining up the digits according to place value. This strategy mimics the way students subtract whole numbers and allows them to solve mixed-number problems quickly. Students also expand their knowledge of the distributive property as they use the property to multiply decimals.

You can expect to see homework that asks your child to do the following:

- Find the sum or difference. (In some cases, students will use a calculator and follow specific instructions for how to round the final answer.)
- Calculate the product by using partial products. (See the second Sample Problem.)
- Solve word problems by adding, subtracting, or multiplying decimals.

SAMPLE PROBLEMS  (From Lessons 9 and 10)

After Arianna completed some work, she still had $78 \frac{21}{100}$ pictures to paint. If she completes $34 \frac{23}{25}$ more pictures, how many pictures will Arianna still have left to paint?

Expression: $78 \frac{21}{100} - 34 \frac{23}{25}$

Estimated answer: $78 - 35 = 43$

Actual answer: $78.21 - 34.92 = 43.29$

Use partial products and the distributive property to calculate the product.

$200 \times 32.6$

$200(32 + 0.6)$

$200(32) + 200(0.6) = 6,400 + 120 = 6,520$

Additional sample problems with detailed answer steps are found in the Eureka Math Homework Helpers books. Learn more at GreatMinds.org.

For more resources, visit » Eureka.support
**HOW YOU CAN HELP AT HOME**

You can help at home in many ways. Here are just a few tips to help you get started.

- Estimate the product of \(500 \times 12.7\) (\(500 \times 10 = 5,000\)). Then, with your child, find the product using partial products. \((500(10 + 2 + 0.7) = 500(10) + 500(2) + 500(0.7) = 5,000 + 1,000 + 350 = 6,350)\)

- Show your child this number sentence: \(11.5 \times 13.5 = 15.525\). Challenge your child to explain why the decimal point in the product (the answer) is in the incorrect place. Your child may say something like, “I can see right away that the decimal is in the wrong place because \(11 \times 13\) would have to be more than 15. In fact, I know \(11 \times 13\) is 143, so, if we move the decimal point one place to the right, 155.25 sounds about right.”

- Ask your child to explain how to use unit language to complete the problem \(\frac{2}{5} \div 4\). The first number, \(\frac{2}{5}\), can be renamed as 4 tenths \((\frac{2}{5} = \frac{4}{10} = 4 \text{ tenths})\). This simplifies the problem: \(\frac{2}{5} \div 4 = 4 \text{ tenths} \div 4 = 1 \text{ tenth} = \frac{1}{10}\).

Here is another example of how unit language can be used to make dividing fractions simpler:

\[
\frac{3}{5} \div \frac{1}{4} = \frac{12}{20} \div \frac{5}{20} = 12 \text{ twentieths} \div 5 \text{ twentieths} = \frac{12}{5} = 2\frac{2}{5}.
\]

**TERMS**

**Difference:** The answer to a subtraction problem.

**Distributive property:** Allows the numbers in a multiplication problem to be broken down into partial products (i.e., partial answers) to make the mental math simpler. The partial products can then be combined to find the end product (the answer to the original multiplication problem). For example, consider the problem \(6 \times 27\). The number 27 can be broken down into \((20 + 7)\), so \(6 \times 27 = (6 \times 20) + (6 \times 7) = 120 + 42 = 162\).

**Factors:** Numbers that are multiplied together to get other numbers. For example, 2 and 3 are factors of 6 because \(2 \times 3 = 6\); 4 and 5 are factors of 20 because \(4 \times 5 = 20\).

**Partial products:** The results when you decompose, or break down, the factors in a multiplication problem according to place value and multiply them. For example, \(64 \times 27 = (60 \times 20) + (60 \times 7) + (4 \times 20) + (4 \times 7)\). Therefore, the partial products of \(67 \times 24\) are 1,200, 420, 80, and 28.

**Product:** The answer to a multiplication problem.

**Standard algorithm:** Step-by-step procedures used to solve a particular type of problem. In this module, students will learn and use the standard algorithms for adding, subtracting, multiplying, and dividing decimals, whole numbers, and fractions.

**Sum:** The answer to an addition problem.
KEY CONCEPT OVERVIEW

In Topic C, students use estimation and the standard algorithm for division (see Sample Problems) to divide whole numbers and decimals. Students begin by working extensively with whole numbers to develop an understanding of each step of the algorithm and why it makes sense. The topic wraps up by extending this learning to division of multi-digit decimals.

You can expect to see homework that asks your child to do the following:

▪ Round to estimate the quotient. Then, use a calculator to compute the exact quotient, and compare the estimate to the exact quotient.
▪ Use mental math, estimation, and/or the division algorithm to divide whole numbers and multi-digit decimals (remembering to create a whole number divisor).
▪ Solve word problems by dividing whole numbers or decimals.
▪ Place the decimal point in the correct place to make a number sentence true.

SAMPLE PROBLEMS (From Lesson 13)

Estimate, and then use the standard algorithm to solve $952,448 \div 112$.

a. Estimate: $1,000,000 \div 100 = 10,000$

b. Standard Algorithm:

```
  8 5 0 4
1 1 2 | 9 5 2 4 4 8
      4
    8 9 6
   5 6 4
    1
  5 6 0
  4 4
  0
```

$952 \text{ thousands} \div 112: 8 \text{ thousands}$

$564 \text{ hundreds} \div 112: 5 \text{ hundreds}$

$44 \text{ tens} \div 112: 0 \text{ tens}$

$448 \text{ ones} \div 112: 4 \text{ ones}$
SAMPLE PROBLEMS  (continued) (From Lesson 14)

In the problem below, first make the divisor a whole number by multiplying both the numerator and denominator by 10. Then divide, and check your answer.

\[
3,581.9 \div 4.9
\]

\[
\begin{array}{c}
\begin{array}{r}
3,581.9 \\
\times 10
\end{array} \\
\hline
4.9 \\
49
\end{array}
\]

\[
\frac{3,581.9 \times 10}{4.9} = \frac{35,819}{49}
\]

\[
\begin{array}{c}
\begin{array}{c}
\overline{49} \\
35819
\end{array} \\
\begin{array}{c}
\begin{array}{c}
\overline{731} \quad 1
\end{array} \\
\begin{array}{c}
\begin{array}{c}
\overline{49} \\
\begin{array}{c}
\overline{731} \\
\begin{array}{c}
\begin{array}{c}
\overline{3} \\
\overline{151}
\end{array}
\end{array}
\begin{array}{c}
\overline{2}
\end{array}
\begin{array}{c}
\overline{147}
\end{array}
\begin{array}{c}
\overline{4}
\end{array}
\begin{array}{c}
\overline{9}
\end{array}
\begin{array}{c}
\overline{0}
\end{array}
\end{array}
\end{array}
\end{array}
\]
\end{array}
\]

Check:

\[
35,819 \div 49 = 731
\]

\[
731 \times 49 = 35,819
\]

\[
3,581.9 \div 4.9 = 731
\]

\[
731 \times 4.9 = 3,581.9
\]

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

- Complete a division problem with your child. First, estimate the answer. Then, take turns completing each step in the standard algorithm to find the actual answer. Compare the real answer to the estimate to be sure your answer makes sense. You can use whole numbers or decimals.

- David estimated 5,000 as the quotient for the problem 99,066 ÷ 19. Does his estimate make sense? With your child, discuss what David’s thought process might have been when determining the estimate. (Your child should understand that David probably rounded the problem to 100,000 ÷ 20. Because this expression equals 5,000, David’s estimate makes sense.)

- Reinforce the importance of estimation. Share some ways you use estimation in the real world. For example, estimate how long it will take you to run a few errands or how much the items in your grocery cart will cost.

TERMS

**Divisible:** When one number can be divided by another and the result (quotient) is an exact whole number, we can say that number is divisible by the other number. For example, 36 is divisible by 9 because 36 ÷ 9 = 4.

**Multiple:** The product of a given number and any other whole number. For example, 5, 10, 15, 20, and 25 are all multiples of 5 because 5 can be multiplied by a whole number to equal each of these numbers.
KEY CONCEPT OVERVIEW

In Topic D, students work with odd and even numbers and the divisibility rules to find factors, multiples, common factors, and common multiples of whole numbers. Then they find the greatest common factor and the least common multiple shared by pairs of numbers. To find the greatest common factor for pairs of large numbers, students explore Euclid’s algorithm. (See Sample Problem.)

You can expect to see homework that asks your child to do the following:
▪ Determine whether a sum or product is even or odd.
▪ Use the divisibility rules to determine whether a number is divisible by other numbers. Find a number that is divisible by other numbers.
▪ Identify factors and multiples of given numbers.
▪ Find the greatest common factor and least common multiple of pairs of numbers.
▪ Use Euclid’s algorithm to find the greatest common factor of a pair of large numbers.

SAMPLE PROBLEM (From Lesson 19)

Apply Euclid’s algorithm to find the greatest common factor of 30 and 45, denoted GCF (30, 45).

\[
\begin{array}{c|c}
30 & 45 \\
\hline
1 & 2 \\
30 & 15 \\
-30 & -30 \\
15 & 00 \\
\end{array}
\]

The greatest common factor of 30 and 45 is 15.

Additional sample problems with detailed answer steps are found in the Eureka Math Homework Helpers books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.
▪ Ask your child to draw pairs of dots to explain why the sum of 11 and 15 is even. (This strategy will be familiar from our work in class.) Your child should explain the following: “There is one ‘leftover’ dot when circling pairs to represent 11 and one leftover dot when circling pairs for 15. These leftovers form another pair. There are no more leftover dots, so the sum is even.”

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• Ask your child to draw dots to explain why the sum of 10 and 15 is odd. Following the above procedure, there is one leftover dot, so the sum is odd.

• Take turns listing the factors of 42 (1, 2, 3, 6, 7, 14, 21, and 42) and 70 (1, 2, 5, 7, 10, 14, 35, and 70). Have your child circle the common factors, put a triangle around the greatest common factor (GCF), which is 14, and explain out loud how she found the GCF. Then challenge her to describe the process in writing. Review your child’s writing and work with her to make one or two improvements (e.g., make the vocabulary more precise, add more details to the steps, make the explanation more concise). Your child may say something like, “I found the greatest common factor by listing the factors of each individual number. Then I found the common factors and put a circle around those numbers. I know the greatest common factor is the largest factor that both numbers have in common, so I looked for the largest common factor and put a triangle around that number.”

**TERMS**

**Common factors:** Factors shared by two or more numbers. For example, 3 is a common factor of 6, 9, and 12.

**Common multiples:** Multiples shared by two or more numbers. For example, 30 is a common multiple of 3, 6, and 10.

**Divisibility rules:** Ways to tell whether one whole number is divisible by another.

**Euclid’s algorithm:** A series of steps for finding the greatest common factor of two large numbers.

**Greatest common factor (GCF):** The largest number that divides evenly into all numbers in a group of two or more numbers. To determine the greatest common factor of two numbers—for example, 12 and 16—list all the whole number factors of 12 (1, 2, 3, 4, 6, 12) and all the whole number factors of 16 (1, 2, 4, 8, 16). The greatest whole number that appears on both lists is 4, so 4 is the greatest common factor of 12 and 16.

**Least common multiple (LCM):** The smallest whole number multiple shared by all numbers in a group of two or more numbers. To find the least common multiple of two numbers—for example, 5 and 6—list the first few multiples of each number: 5, 10, 15, 20, 25, 30, and so on for 5, and 6, 12, 18, 24, 30, and so on for 6. The first (and, thus, least) multiple they share is the least common multiple (30).

**Units digit:** The number in the ones place. For example, the units digit for 2,981 is 1, the units digit for 570 is 0, and the units digit for 19,823.4 is 3.
In Topic A, students are introduced to integers and are asked to determine where they are located on horizontal and vertical number lines. They learn about opposites and the opposite of the opposite. They also use integers to represent real-world situations and describe what the value of zero represents in various contexts. Students wrap up the topic by extending their understanding of integers as they locate rational numbers on the number line.

You can expect to see homework that asks your child to do the following:

- Graph a point and its opposite on the number line.
- Express various situations as integers. For example, a fee of $2 is represented as $−2$.
- Describe what zero represents in a given situation. For example, zero represents no change taking place in a bank account.
- Find the opposite of a number and its location on the number line.
- Write an equation to represent the opposite or opposite of the opposite of a number. For example, the following equation represents the opposite of negative seven: $−(−7) = 7$.

SAMPLE PROBLEM (From Lesson 6)

Use what you know about the point $−\frac{7}{4}$ and its opposite to graph both points on the number line below. The fraction $−\frac{7}{4}$ is located between which two consecutive integers? Explain your reasoning.

On the number line, each segment between tick marks will have an equal length of $\frac{1}{4}$. The fraction $−\frac{7}{4}$ is located between $−1$ and $−2$.

Explanation:

$\frac{7}{4}$ is the opposite of $−\frac{7}{4}$. It is the same distance from zero but on the opposite side of zero. Since $−\frac{7}{4}$ is to the left of zero, $\frac{7}{4}$ is to the right of zero. The original fraction, $−\frac{7}{4}$, is located between $−2$ (or $−\frac{8}{4}$) and $−1$ (or $−\frac{4}{4}$).

Additional sample problems with detailed answer steps are found in the Eureka Math Homework Helpers books. Learn more at GreatMinds.org.
HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

▪ Have your child create a number line from −10 to 10. It may help to use grid paper. State a fractional value (e.g., \(\frac{2}{3}\) or \(\frac{4}{5}\)) or a decimal value (e.g., 1.5 or −7.25), and ask your child to correctly locate that value and its opposite on the number line.

▪ Play Integer War. Use a standard deck of cards, assigning red to represent negative values and black to represent positive values. All red face cards represent −10, and all black face cards represent 10. Shuffle the deck and divide the cards evenly between you and your child. Each of you flips over one card, and the person with the card showing the larger value wins that turn and collects both cards. Continue to play until one player collects all the cards and wins the game.

TERMS

**Integer:** A positive or negative whole number, including the number zero. The set of integers is: {..., −3, −2, −1, 0, 1, 2, 3, ...}.

**Negative number:** A number that is less than zero.

**Opposites:** Numbers that are the same distance from zero on the number line but are on different sides of zero. For example, −3 and 3 are opposites.

**Opposite of the opposite:** A number that has the same value as the original number. The opposite of the opposite of −3 is written as \(−(−(−3))\), which is −3.

**Positive number:** A number that is greater than zero.

**Rational number:** A number that can be written as a ratio or fraction involving two integers, the second of which is not zero (the denominator of a fraction cannot be zero). Rational numbers include integers (e.g., 4 because it can be written as \(\frac{4}{1}\)), fractions, terminating (ending) decimals, and repeating decimals.

MODELS

**Horizontal Number Line**

**Vertical Number Line**
In Topic B, students put integers and other rational numbers in order, locate them on the number line, and compare them. They also write and interpret inequality statements. The topic wraps up by asking students to use absolute value to find the magnitude of a positive or negative number in a real-world situation.

You can expect to see homework that asks your child to do the following:

▪ Put a list of numbers, their opposites, and their absolute values in order.
▪ Identify the numbers that are farthest to the left or farthest to the right on a horizontal number line (highest or lowest on a vertical number line).
▪ Write a story relating integers and other rational numbers to real-life situations.
▪ Write an inequality.
▪ Compare the magnitudes of various numbers.

**SAMPLE PROBLEM** *(From Lesson 13)*

During the summer, Madison monitors the water level in her parents’ swimming pool to make sure it is not too far above or below normal. The table below shows the numbers she recorded in July and August to represent how the water levels compared to normal. Order the rational numbers from least to greatest. Explain why the rational numbers that you chose appropriately reflect the given water levels.

<table>
<thead>
<tr>
<th>Madison’s Readings</th>
<th>$\frac{1}{2}$ inch above normal</th>
<th>$\frac{1}{4}$ inch above normal</th>
<th>$\frac{1}{2}$ inch below normal</th>
<th>$\frac{1}{8}$ inch above normal</th>
<th>$1\frac{1}{4}$ inches below normal</th>
<th>$\frac{3}{8}$ inch below normal</th>
<th>$\frac{3}{4}$ inch below normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compared to Normal</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{4}$</td>
<td>$-\frac{1}{2}$</td>
<td>$\frac{1}{8}$</td>
<td>$-1\frac{1}{4}$</td>
<td>$-\frac{3}{8}$</td>
<td>$-\frac{3}{4}$</td>
</tr>
</tbody>
</table>

$-1\frac{1}{4} < -\frac{3}{4} < -\frac{1}{2} < -\frac{1}{8} < -\frac{1}{4} < \frac{1}{2}$

*The measurements are taken in reference to the normal level, which is considered to be 0. The words above normal refer to the positive numbers located above zero on a vertical number line, and the words below normal refer to the negative numbers located below zero on a vertical number line.*

Additional sample problems with detailed answer steps are found in the Eureka Math Homework Helpers books. Learn more at GreatMinds.org.

For more resources, visit » Eureka.support
HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

▪ Have your child make integer cards by writing integers from $-10$ to $10$ on note cards or flash cards, with one integer per card. Show the cards three at a time, and ask your child to put the integers in order from either least to greatest or greatest to least. When your child is comfortable ordering integers, make additional cards with rational numbers, including fractions and decimals, from $-10$ to $10$. Add these cards to the integer cards, and repeat the activity.

▪ Create a number line from $-10$ to $10$ on the floor. (Tiled floors work great for this activity!) Using the cards from the activity above, choose one card, and have your child locate and stand on that number on the number line. Then, have your child move to that number’s opposite. Discuss how far the number and its opposite are from zero. If a fraction or decimal is chosen, have your child estimate where that number is on the number line. For example, the number $7.4$ would be slightly less than halfway between $7$ and $8$.

TERMS

Absolute value: The distance between a number and zero on the number line, shown symbolically as $|a|$ (e.g., $|3| = 3$ or $|-4| = 4$).

Inequality: A statement comparing expressions that are unequal or not strictly equal. The symbol used to compare the expressions reveals the type of inequality: $<$ (less than), $\leq$ (less than or equal to), $>$ (greater than), $\geq$ (greater than or equal to), or $\neq$ (not equal to).

Magnitude: The absolute value of the number in a measurement, that is, the distance between the number and zero on a number line. For example, the magnitude of the measurement $-25^\circ$F is $25$. 
KEY CONCEPT OVERVIEW

In Topic C, students extend their understanding of the number line to the coordinate plane and identify points in all four quadrants. They locate and label points whose ordered pairs differ only by the sign (positive or negative) of one or both coordinates and recognize symmetry across both axes. For example, the points (2, 7) and (2, −7) represent a reflection across the x-axis. Students also draw and label the coordinate plane, using all necessary components (origin, axes, appropriate scale), and determine the lengths of line segments by counting or by using their understanding of absolute value.

You can expect to see homework that asks your child to do the following:

- Notice relationships between the first and second coordinates in an ordered pair. For example, in the ordered pair (15, 9), the first and second coordinates have a greatest common factor of 3.
- Name the quadrant in which a specific point lies and locate points in specific quadrants.
- Reflect a point over a given axis, label the image, and analyze the relationship between the coordinates in the ordered pair for each point.
- Find the length of a line segment with given endpoints.
- Given the length of a line segment and the ordered pair for one endpoint, determine a possible ordered pair of the other endpoint.
- Given the ordered pairs of two vertices (corner points) in a rectangle and its perimeter, determine the ordered pairs of the other two vertices.

SAMPLE PROBLEMS (From Lessons 16 and 19)

In each column, write the coordinates of the points that are related to the given point by the criteria listed in the first column of the table. Point S (5, 3) has been reflected over the x- and y-axes for you as a guide, and the resulting images are shown on the coordinate plane. Use the coordinate grid to help you locate each point and its corresponding coordinates.

1. When the coordinates of two points are (x, y) and (−x, y), what line of symmetry do the points share? Explain.

   They share the y-axis as their line of symmetry because the y-coordinates are the same and the x-coordinates are opposites. This means the points will be the same distance from the y-axis but on opposite sides.

2. When the coordinates of two points are (x, y) and (x, −y), what line of symmetry do the points share? Explain.

   They share the x-axis as their line of symmetry because the x-coordinates are the same and the y-coordinates are opposites. This means the points will be the same distance from the x-axis but on opposite sides.

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3. On the coordinate plane, locate and label (4, 5) and (4, −3). Draw a line segment to connect the points. How long is the line segment that you drew? Explain.

*The length of the line segment is 8 units. The endpoints are on opposite sides of the x-axis. I added the absolute values of the second coordinates together, so the distance from end to end is 8 units. I could have counted the units, which would also result in a length of 8 units.*

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

**HOW YOU CAN HELP AT HOME**

You can help at home in many ways. Here is a tip to help you get started.

- Make a coordinate grid on the floor. (Tiled floors work great!) Label both axes from −5 to 5. Have your child make 10 ordered-pair cards by writing one ordered pair on a note card or flash card. Choose one card. Have your child find and stand at that location on the grid and state the quadrant where the point lies. (See Models section below.) Then, have your child stand at the location of the point that differs by one sign. For example, if he is at the location (1, 3), he would move to (1, −3) or (−1, 3). Ask him to discuss the similarities and differences in the coordinates. For example, if the ordered pairs are (1, 3) and (1, −3), they have the same x-coordinate but opposite y-coordinates. Then, ask your child to discuss the similarities and differences in the location of the two points. For example, each point is 1 unit to the right of the y-axis and 3 units away from the x-axis. Finally, what line of symmetry do the points share? (See Sample Problems.)

**TERMS**

Coordinate: The location of a point on the coordinate plane, written \((x, y)\). The first number is always the x-value of the point (left/right), and the second number is always the y-value of the point (up/down). In the image in the Models section, \((-3, 1)\) is located 3 units to the left of 0 (along the x-axis) and 1 unit up (along the y-axis).

Line of symmetry: The imaginary line through an image such that, when folded on that line, the two halves are mirror images of each other.

Ordered pair: Two numbers written in a given fixed order, usually as \((x, y)\).

Origin: The point where the two axes intersect in the coordinate plane. Its coordinates are \((0, 0)\).

Quadrant: Any of the four equal areas created by dividing a plane by an x-axis and a y-axis. They are numbered I, II, III, and IV, starting in the top right quadrant and moving counterclockwise. (See image in Models section.)

Reflection: Creates a mirror image of a geometric figure on the opposite side of—and the same distance from—the line of reflection (the x-axis or y-axis). Reflections are also referred to as flips because they flip the image over the line of reflection.

**MODELS**

Coordinate Plane
In Topic A, students use a tape diagram to examine relationships between operations. They begin by exploring the relationship between addition and subtraction. Next, they explore the relationships between multiplication and division and multiplication and addition. Students conclude the topic by exploring how subtraction and division are related.

You can expect to see homework that asks your child to do the following:

- Fill in the missing part of a number sentence or equation.
- Explain why the equations \( w - x + x = w \) and \( w + x - x = w \) are called identities.
- Examine and describe the relationships between operations.
- Write an equivalent expression to show a specific relationship. For example, \( 3 \times 9 \) is equivalent to \( 9 + 9 + 9 \), or \( 3d \) is equivalent to \( d + d + d \).
- From a division equation, write the related subtraction equation, draw the tape diagram, and determine the value of the variable. (See the Sample Problem.)

**SAMPLE PROBLEMS (From Lessons 3-4)**

Write the addition and multiplication expressions that describe the model.

\[
\begin{align*}
\text{5 + 5 + 5 and } 3 \times 5
\end{align*}
\]

Using the equation \( 18 \div x = 3 \), write a related subtraction equation, and represent it as a tape diagram. Then, state the value of \( x \).

\[
\begin{align*}
18 - x - x - x &= 0
\end{align*}
\]

\[
x = 6
\]
HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

▪ Read this statement with your child: “When a number is multiplied and divided by the same number, the result is the original number.” For example, $11 \times 5 \div 5 = 11$. Ask your child to write a few examples to show this. Discuss with your child why the equation $3 \times 9 \div 3 = 9$ does not represent this statement. (A number is not multiplied and then divided by the same number. The 3 is multiplied by 3 and then divided by 3, not 9.)

▪ With your child, create a few examples of real-life money situations where the result is the original number. For example, say that you have $10. You spend $5 at the store. Then you find $5 on the sidewalk. How do your examples relate to the identities $w - x + x = w$ or $w + x - x = w$? ($10 - 5 + 5 = 10$)

TERMS

Equation: A statement indicating that two expressions are equal (e.g., $3 \times 4 = 6 \times 2$ and $5 + x = 20$).

Equivalent expressions: Expressions that have the same value (e.g., $2 \times 6$ is equivalent to $4a$ if $a = 3$).

Expression: A group of numbers, symbols, and operators such as + and − with no equal sign that evaluates to a number (e.g., $2 \times 4$ and $9n + 7$).

Identity: An equation that is true no matter what values are substituted for the variables (e.g., $w - x + x = w$ because $w$ and $x$ can be replaced with any numbers, and the equation would remain true).

Number sentence: A statement indicating that two numerical expressions are equal (e.g., $8 - 2 = 5 + 1$).

Variable: A symbol, such as a letter, that is a placeholder for a number.
KEY CONCEPT OVERVIEW

In Topic B, students extend their knowledge of **exponents** from Grade 5 as they strengthen their understanding of the related vocabulary (**base**, **power**, **exponent**, **cubed**, and **squared**) and move from whole number bases to bases written in fraction and decimal form. After studying exponents, students build on knowledge from Topic A. They learn more about the order of operations and how it is used to **evaluate** various **numerical expressions** by examining operations in terms of how **powerful** they are.

You can expect to see homework that asks your child to do the following:

- Write a number in exponential, expanded, and standard form.
- Explain why a whole number base raised to a whole number exponent gets larger, while a fractional base raised to a whole number exponent gets smaller.
- List all the powers of 3 and 4 that evaluate to any number between 3 and 1,000.
- Describe the advantage of **exponential notation** (rather than a multiplication expression) if all the factors are the same.
- Explain the difference between expressions using their knowledge of exponents. For example, $3x$ and $x^3$ are different because if $x$ has a value of 2, the value of $3x$ is $3 \times 2$, or 6, and the value of $x^3$ is $2 \times 2 \times 2$, or 8.
- Evaluate an expression using the order of operations.

SAMPLE PROBLEM  *(From Lesson 6)*

Evaluate using the order of operations.

$2^4 \cdot (13 + 5 - 14 ÷ (3 + 4))$

$2^4 \cdot (13 + 5 - 14 ÷ 7)$

$2^4 \cdot (13 + 5 - 2)$

$2^4 \cdot 16$

$16 \cdot 16$

$256$

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

For more resources, visit » *Eureka.support*
HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

▪ With your child, evaluate the following expressions (find the answer): \((5 + 3^2) ÷ (3 + 4)\) and \(5 + 3^2 ÷ 3 + 4\). Discuss why the answers are different. (They are different because of the parentheses. The first expression has a value of 2, and the second expression has a value of 12. It is important to pay attention to the placement of the parentheses.)

▪ Jeremy thinks \(2^4\) is equal to 8. Suzie thinks the answer is 16. Discuss with your child who is correct and why. (Suzie is correct because the exponent tells how many times the base is multiplied by itself, \(2 × 2 × 2 × 2\). The exponent and base should not be multiplied by each other. Jeremy’s error is a very common mistake, so make sure your child understands and can articulate the error.)

▪ Where can the parentheses be placed so the expression \(28 − 3 × 3 + 4\) has a value of 7? (Around the \(3 + 4\)) Where can the parentheses be placed so the same expression has a value of 79? (Around the \(28 − 3\)) Tyler added an exponent to a term, and now the expression (with no parentheses) has a value of 35. Where did Tyler put the exponent? (He changed 4 to \(4^2\).)

TERMS

**Algebraic expression:** An expression containing numbers, variables, and operators (such as + and −) that represents a single value and does not contain equal signs or inequality symbols (e.g., \(2m \) or \(9a + 3\)).

**Base:** In the term \(y^6\), the \(y\) is the base, or repeating factor, and may be a variable or a number.

**Cubed:** When a base is raised to the third power. For example, \(5^3\) can be read as **five cubed**.

**Evaluate:** To evaluate an expression means to find the answer.

**Exponential notation for whole number exponents:** A way to write numbers using exponents. For example, the number 3,125 (standard form) can be written as \(5 × 5 × 5 × 5 × 5\) (expanded form) or \(5^5\) (exponential form). It provides a simpler alternative to expanded form when indicating that a number should be multiplied by itself repeatedly. We can read \(5^5\) as **five to the fifth power**.

**Exponent:** In the term \(3y^6\), the \(6\) is the exponent. The exponent tells you how many times to use the base \((y)\) as a factor.

**Numerical expression:** A group of numbers, symbols, and operators (such as + and −) that represents a single value and does not contain equal signs or inequality symbols (e.g., \(2 × 4\) or \(9(5 + 1)\)).

**Squared:** When a base is raised to the second power. For example, \(5^2\) can be read as **five squared**.

**Value of a numerical expression:** The number found by evaluating the expression, or, in other words, by simplifying the expression to a single value. For example, the value of the expression \(3 × 8\) is 24.
KEY CONCEPT OVERVIEW

In Topic C, students replace letters with numbers and numbers with letters. They replace a letter with a specific number in order to evaluate an expression to determine its value. They connect this learning to geometry and find the area, perimeter, and volume of various figures, building on their knowledge of exponents. Students also explore and build identities, laying the foundation for solving equations.

You can expect to see homework that asks your child to do the following:

- Replace the dimensions of a figure with numbers and calculate the area, perimeter, and/or volume. For example, if the side lengths of a rectangle are given as \(a\) and \(b\), replace \(a\) and \(b\) with numbers and calculate the area (\(a \times b\)).
- Write an expression given specific information.
- State the **commutative properties** of addition and multiplication using given variables (e.g., \(a + b = b + a\) and \(a \times b = b \times a\)).
- State the **additive identity property of zero**, using a given variable (e.g., \(a + 0 = a\)).
- State the **multiplicative identity property of one**, using a given variable (e.g., \(a \times 1 = a\)).
- Explain why there is no commutative property for subtraction and division.

SAMPLE PROBLEMS  (From Lessons 7 and 8)

1. Complete the table for both figures. You may use a calculator.

<table>
<thead>
<tr>
<th>Length of Rectangular Prism</th>
<th>Width of Rectangular Prism</th>
<th>Height of Rectangular Prism</th>
<th>Rectangular Prism’s Volume Written as an Expression</th>
<th>Rectangular Prism’s Volume Written as a Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 units</td>
<td>5 units</td>
<td>15 units</td>
<td>12 units (\times) 5 units (\times) 15 units</td>
<td>900 cubic units</td>
</tr>
<tr>
<td>23 cm</td>
<td>4 cm</td>
<td>7 cm</td>
<td>23 cm (\times) 4 cm (\times) 7 cm</td>
<td>644 cm(^3)</td>
</tr>
</tbody>
</table>

For more resources, visit » Eureka.support
SAMPLE PROBLEMS (continued)

2. Replace the 3’s in these number sentences with the letter $a$.

$$3 + 3 + 3 + 3 = 4 \times 3$$

$$3 \div 4 = \frac{3}{4}$$

$$a + a + a + a = 4 \times a$$

$$a \div 4 = \frac{a}{4}$$

Choose a value for $a$, and replace $a$ with that number in the first equation. What do you observe?

*If $a = 5$, then $5 + 5 + 5 + 5 = 4 \times 5$, and the result is a true number sentence.*

Will all values of $a$ result in a true number sentence? Experiment with different values before making your claim.

*Yes, any number, even zero, can be used in place of the variable $a$.*

Additional sample problems with detailed answer steps are found in the Eureka Math Homework Helpers books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

- Create two examples that show why there is no commutative property for subtraction or division. For example, $7 - 5$ does not equal $5 - 7$, and $4 \div 2$ does not equal $2 \div 4$.

- Make flash cards with the four properties learned in this topic: the commutative properties of addition and multiplication, the additive identity property, and the multiplicative identity property. On one side, write the property; on the other side, write two or three examples. With your child, review these properties, encouraging him to explain each property in words.

TERMS

**Additive identity:** By definition, the number zero. (See *additive identity property of zero* below.)

**Additive identity property of zero:** The additive identity (zero) can be added to any number without changing the identity of the number (e.g., $11 + 0 = 11$ and $a + 0 = a$).

**Commutative property:** The order of an addition or multiplication problem may change, but the sum or product will remain the same.

**Multiplicative identity:** By definition, the number one. (See *multiplicative identity property of one* below.)

**Multiplicative identity property of one:** The multiplicative identity (one) can be multiplied by any number without changing the identity of the number (e.g., $4 \times 1 = 4$ and $a \times 1 = a$).
KEY CONCEPT OVERVIEW

In this topic, students use a tape diagram to write addition and subtraction expressions. They identify parts of an expression and write multiplication expressions in various ways: $11 \times a$, $11 \cdot a$, or $11a$. Using knowledge of the greatest common factor (GCF) and the distributive property, students write expressions in factored and expanded forms. To conclude Topic D, students write division expressions in two forms: dividend $\div$ divisor and $\frac{\text{dividend}}{\text{divisor}}$.

You can expect to see homework that asks your child to do the following:

▪ Given an expression in word form, write the expression in standard form. For example, write the sum of $g$ and $5$ as $g + 5$.

▪ Rewrite an expression in standard form, for example, $6 \cdot y$ as $6y$.

▪ Write an expression in factored form, for example, $2x + 8y$ as $2(x + 4y)$.

▪ Find the product of two terms, for example, $8x \cdot 3y = 24xy$.

▪ Use a model to prove two expressions are equivalent.

▪ Use the GCF and the distributive property to write equivalent expressions.

▪ Rewrite a division expression by using words, the division symbol ($\div$), the long division symbol ($\frac{\text{long division}}{\text{symbol}}$), and as a fraction.

SAMPLE PROBLEMS (From Lessons 10 and 13)

1. Write the expression by using the fewest possible symbols and characters. Use math terms to describe both the expression and its parts.

$$2 \times 2 \times 2 \times a \times b$$

8$ab$. The $8$ is the coefficient and a factor, $a$ and $b$ are both variables and factors, and $8ab$ is the product and also a term.

2. Write the expression two ways: with the division symbol and as a fraction.

\[a\text{ divided by } 4\]

\[
\frac{a}{4}\text{ and } a \div 4
\]

Additional sample problems with detailed answer steps are found in the Eureka Math Homework Helpers books. Learn more at GreatMinds.org.
How You Can Help at Home

You can help at home in many ways. Here are some tips to help you get started.

▪ Ask your child to write the problem $3\cdot\overline{m+11}$ in words, as a fraction, and with the division symbol. (The sum of $m$ and 11 divided by 3, $\frac{m+11}{3}$, and $(m + 11) ÷ 3$.) Then, ask your child to evaluate the original expression if $m = 16$. (Answer: 9)

▪ Ask your child to draw a model that shows the expression $w$ increased by 4 (w + 4). Then, write an expression that represents this model $(w + 4)$. Challenge your child to think of another way to write this expression $(4 + w)$ and draw the corresponding model $(4 + w)$.

Terms

Coefficient: A constant factor (not to be confused with a constant) in a variable term. For example, in the term $4m$, 4 is the coefficient, and it is multiplied by the variable $m$.

Term: Part of an expression that can be added to or subtracted from the rest of the expression. In the expression $7g + 8h + 3$, the terms are $7g$, $8h$, and 3.
In this topic, students list various words and phrases used to refer to addition, subtraction, multiplication, and division. For example, addition terms include sum, add, more than, total, and altogether. Students then use the words and phrases they listed to write expressions in various forms. For example, $a - b$ can be written as $a$ minus $b$, the difference of $a$ and $b$, $a$ decreased by $b$, and $b$ subtracted from $a$. Students also write algebraic expressions for given phrases. For example, the phrase triple the sum of $x$ and 17 can be written as $3(x + 17)$.

You can expect to see homework that asks your child to do the following:

- List vocabulary words that can describe a given expression. For example, subtract, difference, triple, and product are words that can be used in reference to the expression $15 - 3x$.
- Write an expression in words to match a given algebraic expression. For example, $x + 2$ can be written as two more than $x$ or the sum of $x$ and 2.
- Write an algebraic expression to match a given phrase. For example, eight minus the product of 2 and $g$ can be written as $8 - 2g$.

SAMPLE PROBLEM (From Lesson 16)

Write an algebraic expression that matches the real-world scenario below. Before writing the expression, underline mathematical terms in the text that you could represent as numbers, variables, or symbols in the expression.

Marcus has 4 more dollars than Yaseen. If $y$ is the amount of money Yaseen has in dollars, write an expression to show how much money Marcus has.

Step 1: Underline mathematical words that could be used to create a symbolic expression.

Marcus has 4 more dollars than Yaseen.

Step 2: Think through the problem. If Yaseen had $7, how much money would Marcus have?

$11

Step 3: How did you get that?

I added 7 and 4.

Step 4: Write an expression to show how much money, in dollars, Marcus has using $y$ to represent the amount of money, in dollars, Yaseen has.

$y + 4$

Additional sample problems with detailed answer steps are found in the Eureka Math Homework Helpers books. Learn more at GreatMinds.org.
HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

▪ Ask your child to write the expressions $d - b^3$ and $\frac{3}{d + f}$ using words. First, brainstorm some mathematical words that describe the operations involved (e.g., sum, difference, cubed, quotient, increased by). Then, write at least one possible way to represent the expression in words. If your child is up for a challenge, have her write more than one version of the expression in words. There are multiple correct answers. For the first expression, your child might suggest “$d$ minus $b$ cubed” or “the difference of $d$ and the quantity of $b$ to the third power.” For the second, your child might suggest “the quotient of 3 and the sum of $d$ and $f$” or “3 divided by the quantity $d$ plus $f$.”

▪ Write an algebraic expression using variables and numbers to represent the following statement: I had $c$ pieces of candy. I ate 3 pieces. I split the remaining pieces equally among two friends. A possible answer is $(c - 3) ÷ 2$. 
KEY CONCEPT OVERVIEW

In this topic, students write and evaluate algebraic expressions involving all four operations—addition, subtraction, multiplication, and division—as well as exponents. Working with word problems, students determine and define the variable. Students also substitute a given value (number) for the variable in order to evaluate the expression. For example, given the phrase “Nathalia wrote 3 more stories than Alan,” students can determine the variable, \( s \), and define \( s \) as the number of stories Alan wrote. The expression that represents how many stories Nathalia wrote is \( s + 3 \). If Alan wrote 4 stories, students can substitute 4 for \( s \), resulting in the expression \( 4 + 3 \). Therefore, Nathalia wrote 7 stories.

You can expect to see homework that asks your child to do the following:

- Read a word problem and identify and define the unknown quantity, or variable. Write an addition or subtraction expression that matches the problem, and then evaluate the expression when given a value for the variable.
- Given information, create a table to show the relationship between two quantities. Analyze the data, noticing patterns, and then write the expression that shows this relationship. Finally, evaluate the expression given values for the variables.

SAMPLE PROBLEMS  (From Lesson 19)

Noah and Carter are collecting box tops for their school. They each bring in one box top per day, starting on the first day of school. However, Carter had a head start because his aunt sent him 15 box tops before school began. Noah’s grandma saved 10 box tops, and Noah added those on his first day.

a. Fill in the missing values that indicate the total number of box tops each boy brought to school.

<table>
<thead>
<tr>
<th>School Day</th>
<th>Number of Box Tops Noah Has</th>
<th>Number of Box Tops Carter Has</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>17</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>19</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>

b. If \( D \) represents the number of days since the new school year began, how many box tops will Noah have brought to school on day \( D \)?

\( D + 10 \)

c. On day \( D \) of school, how many box tops will Carter have brought to school?

\( D + 15 \)
d. On day 10 of school, how many box tops will Noah have brought to school?

\[ 10 + 10 = 20. \text{ On day 10, Noah will have brought in 20 box tops.} \]

e. On day 10 of school, how many box tops will Carter have brought to school?

\[ 10 + 15 = 25. \text{ On day 10, Carter will have brought in 25 box tops.} \]

Additional sample problems with detailed answer steps are found in the Eureka Math Homework Helpers books. Learn more at GreatMinds.org.

**HOW YOU CAN HELP AT HOME**

You can help at home in many ways. Here are some tips to help you get started.

- Darcy charges $9 for each lawn she mows. With your child, write an expression describing Darcy’s earnings for mowing \( m \) lawns \((9m)\). How much will Darcy earn if she mows 2, 3, 4, 6, 10, and 15 lawns? (She will earn $18, $27, $36, $54, $90, and $135, respectively.)

- With your child, create a situation that can be described by the expression \( 7x + 15 \). For example, Julia charges $7 an hour for babysitting services and $15 for gas to and from the client’s house. Define the variable (i.e., what does \( x \) mean?), choose a value for \( x \), and evaluate the expression. What does that value mean in the context of the situation? In the example described, let \( x \) represent the number of hours Julia babysat, and let \( x \) have a value of 3. Then \( 7 \cdot 3 + 15 \) is the expression that tells how much Julia earned. She earned $36 in 3 hours.
In this topic, students learn the important relationship between a number sentence and an equation as they explore the role of the number sentence in finding the \textbf{solution of an equation}. Students identify the value that makes an equation true by substituting that value for the variable and determining whether the resulting number sentence is true. Students also extend their knowledge of the equality (=) and \textbf{inequality} symbols (<, >, ≤, and ≥) and identify various values that make an inequality true or false. Finally, students use the tape diagram to solve one-step equations involving all four operations and then extend this learning to solve two-step and multi-step problems.

You can expect to see homework that asks your child to do the following:

- Replace the variable in an equation or inequality with a given value and determine whether the resulting number sentence is true or false. For example, in the inequality \(4 > 1 + g\), if \(g\) has a value of 3, the resulting number sentence \((4 > 1 + 3)\) is false because 4 is equal to (not greater than) 4. If \(g\) is any number less than 3, then the inequality is true.

- State when equations and inequalities will be true and when they will be false. For example, in the equation \(36 = 9k\), the resulting number sentence is true if \(k = 4\) because \(36 = 9(4)\). If \(k = 3\), then the resulting number sentence is false because \(9(3)\) does not equal 36. In fact, if \(k\) has any value other than 4, then the resulting number sentence will be false.

- Solve an equation using a tape diagram as well as algebraically. (See Sample Problem.)

\textbf{SAMPLE PROBLEM} \hspace{1em} \textit{(From Lesson 26)}

Solve the equation. Use a tape diagram and also solve algebraically. Use substitution (i.e., replace the variable with a value) to check your answer.

\[
12 = 8 + c
\]

**Tape Diagram**

```
<table>
<thead>
<tr>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>c</td>
</tr>
</tbody>
</table>
```

**Algebraically**

\[
12 = 8 + c
\]

\[
12 - 8 = 8 + c - 8
\]

\[
4 = c
\]

\textit{Check: Substitute 4 for c to determine whether the number sentence is true. The number sentence 12 = 8 + 4 is a true number sentence because solving the right side of the equation results in 12 = 12, which is true. So 4 is the correct solution.}

Additional sample problems with detailed answer steps are found in the Eureka Math Homework Helpers books. Learn more at GreatMinds.org.

For more resources, visit » Eureka.support
HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

▪ Given the equation $2 = \frac{h}{7}$, determine the value of the variable $h$ with your child. Ask him to solve this problem with a tape diagram as well as algebraically.

<table>
<thead>
<tr>
<th>Tape Diagram</th>
<th>Algebraically</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{h}{7}$</td>
<td>$2 = \frac{h}{7}$</td>
</tr>
<tr>
<td>2</td>
<td>$2 \cdot 7 = \frac{h}{7} \cdot 7$</td>
</tr>
<tr>
<td>$h$</td>
<td>$14 = h$</td>
</tr>
<tr>
<td>$h \div 7$, $h \div 7$, $h \div 7$, $h \div 7$, $h \div 7$, $h \div 7$</td>
<td></td>
</tr>
<tr>
<td>2, 2, 2, 2, 2, 2</td>
<td>Check:</td>
</tr>
<tr>
<td>$2 = \frac{14}{7}$</td>
<td></td>
</tr>
<tr>
<td>$2 = 2$</td>
<td></td>
</tr>
</tbody>
</table>

Ask your child to explain how he knows that 14 is the correct answer. (The value of the variable is 14 because 14 makes the number sentence true, so it is the solution to the equation.)

▪ With your child, work with the inequalities $12 > 4 + g$, $27 < x - 5.5$, and $11 \leq k + 3$. Take turns stating one number that could make each inequality true. After each of you has taken two turns, work with your child to think of two numbers per inequality that will make each inequality false. Encourage your child to explain why each number you identified makes the inequality true or false.

TERMS

**Inequality:** An inequality is a statement comparing expressions that are unequal or not strictly equal. The symbol used to compare the expressions reveals the type of inequality: $<$ (less than), $\leq$ (less than or equal to), $>$ (greater than), $\geq$ (greater than or equal to), or $\neq$ (not equal).

**Solution of an equation:** For an equation with one variable, the solution is any number you can substitute for the variable to make a true number sentence. For example, in $3x = 24$, the solution of the equation is 8 because $3(8) = 24$.

**Truth values of a number sentence:** The truth value is either true (if the number sentence is true) or false (if the number sentence is false). A number sentence is true when it is mathematically correct. For example, $4 \times 5 + 1 = 21$ is true because $4 \times 5 + 1$ is equal to 21, while $4 + 5 > 11$ is false because $4 + 5$ is not greater than 11.
KEY CONCEPT OVERVIEW

In this topic, students apply their knowledge of solving equations to real-world situations. Using knowledge of angle relationships (e.g., a right angle has a measure of 90 degrees, and a straight angle has a measure of 180 degrees), students write and solve one-step equations to find the unknown measure of an angle. Given a real-world situation, students write an equation with two variables (e.g., \( t = 7m \)), analyze the relationship between the independent and dependent variables, create a table, and plot the points on the coordinate plane. To wrap up the module, students use their understanding of true and false number sentences to write and graph inequalities on a number line diagram.

You can expect to see homework that asks your child to do the following:

- Write an equation to solve for the unknown measure of an angle.
- Identify the independent and dependent variables in a context, write an equation, complete a table, and plot the points from the table on a graph.
- From a set of numbers, choose the number(s), if any, that make a given equation or inequality true.
- Given a phrase (e.g., at least 13), write and graph an inequality (e.g., \( x \geq 13 \)).

SAMPLE PROBLEMS (From Lessons 30 and 32)

1. Write an equation that represents the following situation and solve.

\( \angle ABC \) measures 90°. It has been split into two angles, \( \angle ABD \) and \( \angle DBC \). The measures of the two angles are in a ratio of 2:1. What is the measure of each angle?

*Let \( x^\circ \) represent the measure of \( \angle DBC \).*

\[
\begin{align*}
    x^\circ + 2x^\circ &= 90^\circ \\
    3x^\circ &= 90^\circ \\
    3x^\circ ÷ 3 &= 90^\circ ÷ 3 \\
    x^\circ &= 30^\circ
\end{align*}
\]

*The smaller angle (\( \angle DBC \)) measures 30°. Since the ratio of angle measures is 2:1, the measure of the larger angle (\( \angle ABD \)) has a value of 60° because 30 × 2 = 60.*

2. Each week, Quentin saves $30. Write an equation that represents the relationship between the number of weeks that Quentin has saved his money, \( w \), and the total amount of money in dollars he has saved, \( s \). Then, name the independent and dependent variables. Create a table and a graph that show the total amount of money Quentin has saved from week 1 through week 8. Finally, write a sentence that explains this relationship.

\( s = 30w \)

*The amount of money saved in dollars, \( s \), is the dependent variable, and the number of weeks, \( w \), is the independent variable.*
**SAMPLE PROBLEM** (continued)

<table>
<thead>
<tr>
<th>Number of Weeks</th>
<th>Total Saved ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
</tr>
<tr>
<td>4</td>
<td>120</td>
</tr>
<tr>
<td>5</td>
<td>150</td>
</tr>
<tr>
<td>6</td>
<td>180</td>
</tr>
<tr>
<td>7</td>
<td>210</td>
</tr>
<tr>
<td>8</td>
<td>240</td>
</tr>
</tbody>
</table>

**Therefore, the amount of money Quentin has saved increases by $30 for every week he saves money.**

Additional sample problems with detailed answer steps are found in the Eureka Math Homework Helpers books. Learn more at GreatMinds.org.

**HOW YOU CAN HELP AT HOME**

You can help at home in many ways. Here are some tips to help you get started.

- Encourage your child to identify which number(s) make each inequality true. Given the set of numbers \{3, 4, 9, 12, 24\}, choose the number(s) that make each inequality true.
  
  a. \(m + 7 < 12\) (solution: \{3, 4\})
  
  b. \(t - 2 \leq 9\) (solution: \{3, 4, 9\})
  
  c. \(\frac{k}{3} \geq 2.25\) (solution: \{9, 12, 24\})

- With your child, write three equations that have a solution of \(x = 12\).
  
  (Possible equations: \(24 = 2x\), \(8 = x - 4\), and \(18 = x + 6\).) Then, each of you create an equation for which the solution is a positive whole number between 50 and 100. Exchange equations with your child. Solve each other’s equations, and explain why the solution is correct.

**TERMS**

**Dependent variable:** A variable whose value depends on the value of another variable. For example, if \(x\) represents the number of hours spent studying and \(y\) represents the test score, the value of \(y\) might change according to the value of \(x\).

**Independent variable:** A variable (e.g., age) whose value is not affected by the values of other variables.

**MODELS**

**Graphing Inequalities**
KEY CONCEPT OVERVIEW

In this topic, students relate the parallelogram to the more familiar rectangle and discover that the areas of both figures are calculated the same way (area = base × height, or \( A = bh \)). Students also discover that a right triangle is exactly half of a rectangle. From there, students generalize that the area of every triangle is one-half the area of its corresponding parallelogram. They then generalize the formula for the area of a triangle as \( A = \frac{1}{2}bh \). Using their knowledge of the area formulas for rectangles, triangles, and parallelograms, students find the areas of irregularly shaped polygons (or composite figures) by composing or decomposing the figures into familiar shapes and finding their areas.

You can expect to see homework that asks your child to do the following:
- Draw and label the altitude, or height, of a parallelogram.
- Calculate the areas of rectangles, parallelograms, and triangles using the area formula for each figure.
- Compare the areas of two different figures to see whether they are the same.
- Through decomposition and composition, calculate the area of a composite figure given the lengths of its sides.
- Use knowledge of area to solve real-world problems.

SAMPLE PROBLEMS (From Lesson 3)

Examine the given triangle and expression.

Explain what each part of the expression represents according to the triangle.

\[ \frac{1}{2} \times (11 \text{ ft.})(4 \text{ ft.}) \]

a. The \( \frac{1}{2} \) is **used in the expression because the area of a triangle is half the area of its corresponding parallelogram.**

- **The length of the base of the triangle is 11 ft. because 8 ft. + 3 ft. = 11 ft.**
- **The height of the triangle is 4 ft. because that is the length of the altitude, the perpendicular segment from a vertex to the base.**

b. Joe found the area of the triangle by writing \( A = \frac{1}{2}(11 \text{ ft.})(4 \text{ ft.}) \), while Kaitlyn found the area by writing \( A = \frac{1}{2}(3 \text{ ft.})(4 \text{ ft.}) + \frac{1}{2}(8 \text{ ft.})(4 \text{ ft.}) \). They are both correct. Explain how each student approached the problem.

Joe combined the lengths of each part of the base of the triangle first and then calculated the area of the entire triangle, whereas Kaitlyn decomposed the figure into two smaller right triangles, calculated each area, and then added these areas together.

Additional sample problems with detailed answer steps are found in the Eureka Math Homework Helpers books. Learn more at GreatMinds.org.
HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

▪ Have your child sketch two different rectangles, two different parallelograms, and two different right triangles, each with an area of 16 cm$^2$. Remember to label the dimensions (base and height) of each figure.

▪ Josiah, Jeremiah, and Ryanne were asked to calculate the area of the triangle below.

Their responses were as follows. Josiah stated, “The area of the triangle is 24 in$^2$ because I added the lengths of the sides and I know area is in square units.” Jeremiah stated, “I know area is found by multiplying the length of the base and the length of the height, so the area is 8 in. $\times$ 6 in., or 48 in$^2$.” Ryanne stated, “Since this is a triangle, I multiplied the length of the base and the length of the height and then divided the product by 2. The area is 24 in$^2$.” With your child, figure out who explained the calculation for the area of this triangle correctly. (Ryanne is correct because she used the correct method of finding the area of a triangle: $A = \frac{1}{2}bh$, or $A = (bh) \div 2$. Although Josiah found the correct area, his method was faulty.)

TERMS

Altitude: A perpendicular line segment from the vertex of a triangle to the opposite side (the base). In a parallelogram, the altitude is a perpendicular line segment from the base to its opposite side. The measurement of this line segment is the height of the figure. (See Figures 1 and 3.)

Area: The number of square units (e.g., square feet) that make up the inside of a two-dimensional shape.

Base: The side of a figure that is perpendicular to the altitude. (See Figures 1 and 3.)

Composite figure: A figure that can be divided into more than one basic shape. The trapezoid is composed of a square and a triangle. (See Figure 2.)

Parallelogram: A four-sided closed figure with opposite sides that are parallel and equal in length. (See Figure 3.)

Polygon: A closed shape with straight sides (e.g., triangle, square, rectangle, parallelogram, hexagon).
In this topic, students use absolute value to find the distance between integers in the coordinate plane and to determine the side lengths of polygons. Then, students use the side lengths to calculate the areas of various polygons by decomposing or composing them into shapes with known area formulas. Students also find the perimeters of various polygons in the coordinate plane. In the final lesson of the topic, students apply their knowledge of distance, area, and perimeter to real-world situations.

You can expect to see homework that asks your child to do the following:

- Determine whether the line segment joining two points is horizontal, vertical, or neither.
- Use absolute value to determine the lengths of line segments.
- Name two points that, when connected, form a line segment with a specified length.
- Plot points in the coordinate plane to make a shape, and then find the area and/or perimeter of the shape.
- Calculate and compare the areas and perimeters of various figures.
- Solve real-world math problems involving distance, area, and perimeter.

**SAMPLE PROBLEM**  *(From Lesson 9)*

Jasjeet has made a scale drawing of a vegetable garden she plans to make in her backyard. She needs to determine the perimeter and area to know how much fencing and soil to purchase. Determine both the perimeter and area.

\[ \text{Perimeter} = 4 \text{ units} + 7 \text{ units} + 4 \text{ units} + 6 \text{ units} + 8 \text{ units} + 13 \text{ units} \]

\[ \text{Perimeter} = 42 \text{ units} \]

The area is found by making a horizontal cut from \((1, 1)\) to point \(C\) to decompose into two rectangles.

<table>
<thead>
<tr>
<th>Area of Top</th>
<th>Area of Bottom</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A = lw )</td>
<td>( A = lw )</td>
</tr>
<tr>
<td>( A = (4 \text{ units})(7 \text{ units}) )</td>
<td>( A = (8 \text{ units})(6 \text{ units}) )</td>
</tr>
<tr>
<td>( A = 28 \text{ units}^2 )</td>
<td>( A = 48 \text{ units}^2 )</td>
</tr>
</tbody>
</table>

\[ \text{Total Area} = 28 \text{ units}^2 + 48 \text{ units}^2 \]

\[ \text{Total Area} = 76 \text{ units}^2 \]

Additional sample problems with detailed answer steps are found in the Eureka Math Homework Helpers books. Learn more at GreatMinds.org.
HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

▪ Create a coordinate grid on a piece of graph paper. Plot the points (1, 4) and (1, −7), and draw a line to connect them. Have your child find the distance between the two points (11 units, or 11 squares, on the graph paper). If your child begins counting the units to find the distance, ask her instead to use absolute value and explain her thinking. (Possible solution for total: “Since the y-coordinates, 4 and −7, have different signs—positive and negative—I will add their absolute values, |4| + |−7| = 4 + 7 = 11. So the distance between the two points is 11 units.”)

▪ With your child, find the area and perimeter of this figure by using the strategy of your choice (Area: 130 units², Perimeter: 62 units). Then challenge your child to use a different strategy to determine the area.

(Possible solutions: The figure may be decomposed the following ways: Then your child could find the total area by finding the sum of the areas of all the parts.)

TERMS

Perimeter: The distance around a two-dimensional shape.

MODELS

Coordinate Plane
**KEY CONCEPT OVERVIEW**

In this topic, students apply their knowledge of the formula for volume of a right rectangular prism (Volume = length × width × height, or \( V = lwh \)) to calculate the volumes of right rectangular prisms with fractional side lengths. They pay attention to the units and record volume in cubic units. Later in the topic, students explore another way to find the volume of a right rectangular prism using the formula Volume = area of the base × height (\( V = Bh \)), and they apply both volume formulas to various problems. The topic wraps up with students calculating the volumes of composite solid figures. They use one of the volume formulas to find the volume or unknown dimension in real-world situations.

You can expect to see homework that asks your child to do the following:

- Calculate the volume of a right rectangular prism using the formulas \( V = lwh \) and \( V = Bh \).
- Write numerical expressions to represent the volume of a right rectangular prism in different ways, and explain how those expressions are the same.
- Given the area of the base, calculate the volume of a right rectangular prism for various values of the height.
- Given the volume and the height, write and solve an equation to determine the area of the base of a right rectangular prism.
- Given a figure and its dimensions, calculate the volume if the dimensions are changed (e.g., cut in half).
- Describe how volume changes as the length of a prism changes by a specified amount (e.g., one-third or three times as long).
- Determine the volume of a composite figure.

**SAMPLE PROBLEMS** *(From Lesson 11)*

Use the prism in the diagram at the right to answer the following questions.

a. Calculate the volume.

\[
V = lwh
\]

\[
V = \left(5 \frac{1}{3} \text{ cm}\right) \left(2 \frac{2}{3} \text{ cm}\right) \left(1 \frac{1}{3} \text{ cm}\right)
\]

\[
V = \left(16 \frac{2}{3} \text{ cm}\right) \left(2 \frac{2}{3} \text{ cm}\right) \left(4 \frac{1}{3} \text{ cm}\right)
\]

\[
V = \frac{128}{27} \text{ cm}^3 \text{ or } V = \frac{4}{27} \text{ cm}^3
\]

b. If you have to fill the prism with cubes whose side lengths are less than 1 cm, what size would be best?

*The best choice would be a cube with side lengths of \( \frac{1}{3} \) cm.*
c. How many of the cubes would fit in the prism?

16 × 2 × 4 = 128, so 128 cubes will fit in the prism.

d. Use the relationship between the number of cubes and the volume to verify your volume calculation.

The volume of one cube is \( \left( \frac{1}{3} \text{ cm} \right) \left( \frac{1}{3} \text{ cm} \right) \left( \frac{1}{3} \text{ cm} \right) = \frac{1}{27} \text{ cm}^3 \).

Since there are 128 cubes, the volume is \( 128 \times \frac{1}{27} \text{ cm}^3 = \frac{128}{27} \text{ cm}^3 \), or \( 4 \frac{20}{27} \text{ cm}^3 \), which matches the answer found previously.

Additional sample problems with detailed answer steps are found in the Eureka Math Homework Helpers books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

- Discuss this problem with your child: How many squares with \( \frac{1}{2} \)-unit side lengths can fit inside a square with 1-unit side lengths? Encourage your child to explain (using words or drawings) why 4 squares can fit inside and not just 2, which is a common misconception. (Possible solution: The area of the larger square is 1 square unit. The area of the smaller square with \( \frac{1}{2} \)-unit side lengths is \( \frac{1}{4} \) square unit, that is, \( \frac{1}{4} \) of the area of a square with 1-unit side lengths. So, 4 of the smaller squares will fit inside the larger square.)

- The ability to multiply fractions and mixed numbers efficiently is very important for the work in this topic. Ask your child to determine and simplify the products of the fractions or mixed numbers in the table at the right.

| \( \frac{3}{8} \times \frac{4}{5} \) | \( \frac{12}{40} = \frac{3}{10} \) |
| \( \frac{6}{11} \times \frac{2}{15} \) | \( \frac{12}{66} = \frac{4}{22} = \frac{2}{11} \) |
| \( \frac{12}{3} \times \frac{3}{5} \) | \( \frac{15}{15} = 1 \) |
| \( \frac{2}{3} \times \frac{3}{4} \) | \( \frac{39}{24} = \frac{15}{8} = \frac{3}{5} \) |
| \( \frac{12}{5} \times \frac{2}{3} \) | \( \frac{77}{52} \) |

TERMS

Right rectangular prism: A three-dimensional solid shape with six faces that are all rectangles. (See Image.)

Volume: The amount of space inside a three-dimensional object, such as a cube or prism, measured in cubic units.
KEY CONCEPT OVERVIEW

In this topic, students use nets to create three-dimensional (solid) figures. They identify the net that matches the corresponding solid figure (prism or pyramid). They also construct the net for a particular figure, given its dimensions. The topic wraps up as students extend their knowledge of nets to find the surface areas and volumes of three-dimensional figures.

You can expect to see homework that asks your child to do the following:

- Match a net to the picture of its solid and write the name of the solid.
- Sketch various nets that can fold into a cube.
- Given a net, classify the solid as a prism (two bases, rectangular side faces) or a pyramid (one base, triangular side faces that meet at a vertex) and write the name of the solid (e.g., cube or rectangular pyramid).
- Given a figure or its dimensions, sketch and label the net and the edge lengths.
- Given a net, name the corresponding solid, and then write and evaluate the expression for surface area. (See Sample Problems.)
- Use the formula \(SA = 2lw + 2lh + 2wh\) to calculate the surface area of a right rectangular prism and the formula \(SA = 6s^2\) to calculate the surface area of a cube.
- Solve real-world problems involving surface area and volume.

SAMPLE PROBLEMS  (From Lessons 17 and 18)

1. Name the solid the net would create, write an expression for surface area, and evaluate. Assume that each box on the grid paper represents a 1 cm \(\times\) 1 cm square. Explain how the expression represents the figure.

   **Name of shape: rectangular pyramid**

   **Surface area expression:** \((3 \text{ cm} \times 4 \text{ cm}) + 2 \left( \frac{1}{2} \times 4 \text{ cm} \times 4 \text{ cm} \right) + 2 \left( \frac{1}{2} \times 4 \text{ cm} \times 3 \text{ cm} \right)\)

   **Evaluation:** \(12 \text{ cm}^2 + 2(8 \text{ cm}^2) + 2(6 \text{ cm}^2) = 40 \text{ cm}^2\)

   The surface area is 40 cm\(^2\). The figure has 1 rectangular base that measures 3 cm \(\times\) 4 cm, 2 triangular faces with bases of 4 cm and heights of 4 cm, and 2 other triangular faces with bases of 3 cm and heights of 4 cm.

2. The Quincy Place housing development plans to add a full-sized neighborhood pool. In preparing the budget, Quincy Place determined that it is also possible to install a baby pool requiring less than 15 cubic feet of water. Quincy Place has three different baby pool models to choose from.

   Choice one: 5 ft. \(\times\) 5 ft. \(\times\) 1 ft.
   Choice two: 4 ft. \(\times\) 3 ft. \(\times\) 1 ft.
   Choice three: 4 ft. \(\times\) 2 ft. \(\times\) 2 ft.

Which baby pool is best? Why are the others not good choices?

   **Choice one volume:** 5 ft. \(\times\) 5 ft. \(\times\) 1 ft. = 25 ft\(^3\)
   **Choice two volume:** 4 ft. \(\times\) 3 ft. \(\times\) 1 ft. = 12 ft\(^3\)
   **Choice three volume:** 4 ft. \(\times\) 2 ft. \(\times\) 2 ft. = 16 ft\(^3\)

   **Choice two is within the budget because it holds less than 15 cubic feet of water. The other two choices require larger volumes than Quincy Place can afford.**

Additional sample problems with detailed answer steps are found in the Eureka Math Homework Helpers books. Learn more at GreatMinds.org.

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HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

▪ With your child, sketch three different nets for a cube. (There are 11 possible nets, but Figure 1 shows a few correct answers.)

![Figure 1]

▪ Discuss with your child why Figure 2 is not a net for a cube. (The net shown does not represent a cube because all of the faces are not squares. This net is for a right rectangular prism.)

![Figure 2]

▪ Byron identified the length, width, and height of Figure 3 as 12 in., 2 in., and 3 in., respectively.

![Figure 3]

Ask your child to use Byron’s dimensions to calculate the surface area of the figure using the formula

\[
SA = 2lw + 2lh + 2wh. \quad (SA = 2(12 \text{ in.})(2 \text{ in.}) + 2(12 \text{ in.})(3 \text{ in.}) + 2(2 \text{ in.})(3 \text{ in.}) = 132 \text{ in.}^2) \]

Then point out that the equation \(SA = 2(2 \text{ in.})(12 \text{ in.}) + 2(2 \text{ in.})(3 \text{ in.}) + 2(12 \text{ in.})(3 \text{ in.})\) results in the same answer for the surface area. Ask your child to explain why. (The length, width, and height were identified as 12 in., 2 in., and 3 in., respectively, but the answer is still correct because it also combines the areas of all six sides.)

TERMS

**Net:** The flat, two-dimensional figure that can be folded to form a three-dimensional figure. (See Sample Problem 1 and first bullet in How You Can Help At Home.)

**Rectangular pyramid:** A three-dimensional shape that has a rectangular base and triangular faces that meet at the apex, which is the point, or vertex, farthest from the base.

**Surface area:** The measure of the total area occupied by the surface (outside) of a three-dimensional object. It is measured in square units.

**Surface of a prism or pyramid:** The collection of all the faces of a prism or pyramid. (One face is shown in yellow to the right.) For example, the five faces of a triangular prism form its surface.
In this topic, students begin the study of statistics and use data to answer questions. Students learn to recognize a statistical question and the type of data (categorical or numerical) that are collected to answer it. To organize and summarize the data they collect, students create histograms (for numerical data) and dot plots, noting the advantages and disadvantages of both types of graphs. Students also explore the shape of the data distribution (how it looks on a graph) to determine whether the distribution is symmetric or skewed. (See Models.) In the final lesson of the topic, students extend their knowledge to relative frequency histograms where the vertical scale is relative frequency, not frequency.

You can expect to see homework that asks your child to do the following:

- Determine whether a question is a statistical question and explain his reasoning. If it is not, rewrite it as a statistical question.
- Classify data as categorical or numerical.
- Create a dot plot to represent given data and use the data to answer questions.
- Match a statistical question to the dot plot representing data that answer the question.
- Complete a frequency table; then create a histogram with its data.
- Use a histogram and relative frequency histogram to answer questions. (See Sample Problems.)

**SAMPLE PROBLEMS  (From Lesson 5)**

Below is a relative frequency table of the seating capacity of NBA basketball arenas.

<table>
<thead>
<tr>
<th>Number of Seats</th>
<th>Tally</th>
<th>Frequency</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>17,000–17,500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17,500–18,000</td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>18,000–18,500</td>
<td>+++</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>18,500–19,000</td>
<td>+++</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>19,000–19,500</td>
<td>+++</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>19,500–20,000</td>
<td>+++</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>20,000–20,500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20,500–21,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21,000–21,500</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>21,500–22,000</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>22,000–22,500</td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

a. What is the total number of NBA arenas?

*I added the values in the frequency column, and there are 29 NBA arenas in total.*

b. Complete the relative frequency column. Round the relative frequencies to the nearest thousandth.

*(See the last column in the table above.)*

c. Construct a relative frequency histogram of the arena capacities.

*(See the image to the right.)*

d. Describe the shape of the relative frequency histogram.

*The shape is skewed slightly to the right.*
e. What percentage of the arenas have a seating capacity between 18,500 and 19,999?

**Approximately 51.6% of the arenas have a seating capacity between 18,500 and 19,999.**

Additional sample problems with detailed answer steps are found in the Eureka Math Homework Helpers books. Learn more at GreatMinds.org.

**HOW YOU CAN HELP AT HOME**

You can help at home in many ways. Here are some tips to help you get started.

- Ask your child to write one statistical question and one question that is not statistical and explain the difference. Your child might consider questions such as, “What are the favorite colors of sixth graders in my school?” and “What is my favorite color?” The first question is a statistical question because the favorite color would not be the same for every student, so there would be variability in the data.

- Write a list of any 15 numbers from 6 to 25, with 4 of the numbers repeating at least once. Have your child create a dot plot (see Models) to represent the data and then answer the following questions: “Which number occurs most/least often in the data set? What number would you use to describe the center of the data?”

**TERMS**

**Categorical data:** Data that can be represented as a group or category (e.g., hair color or flavor of ice cream).

**Data set:** A collection of numbers, values, or categorical data often gathered to answer a particular statistical question.

**Frequency:** The number of data values included in each interval displayed in a frequency table or histogram.

**Interval:** A set of numbers that lie between two specific values and include the lower of the two values but not the upper one. The upper value belongs to the next interval.

**Numerical data:** Data that can be represented as numbers (e.g., age or number of pencils).

**Relative frequency:** The number of data values included in each interval divided by the total number of values included in the data set.

**Statistical question:** A question that can be answered by collecting data and that anticipates variability in the data collected.

**Variability:** The extent to which the values in a data set differ from each other; variability occurs when the observations in a data set are not all the same. For example, the variability of the data set {0, 2, 4, 4, 5, 9, 18} is greater than the variability of the data set {2, 3, 3, 3, 3, 4}.

**MODELS**

- **Dot Plot**
- **Skewed Data Distributions**
- **Symmetric Data Distribution**
This topic focuses on data sets that are approximately symmetric (mound-shaped). The **mean** is the measure of center and the **mean absolute deviation (MAD)** is the measure of variability that describes these data sets. Students explore the “fair share” understanding of mean and also interpret mean as a balance point (see Terms), which helps students build a foundation for calculating the MAD. Students also use graphs (dot plots or histograms) as well as the numerical summaries (mean and MAD) to describe, compare, and answer questions regarding data sets.

You can expect to see homework that asks your child to do the following:

- Draw a representation of a data set by using cubes, where one cube represents one value.
- Use the fair share method to find the mean, shifting cubes between stacks to distribute them evenly.
- Draw and describe a dot plot that represents a particular data set.
- Find the mean by adding all the values and then dividing the sum by the number of values.
- Find the sum of the **absolute deviations**.
- After analyzing two data sets, recognize which mean is a better representation of the data set. Also, recognize which data set has the larger MAD and explain.
- Calculate the MAD of a data set.
- Given the mean and MAD, draw a dot plot.
- Given data and a real-world context, draw a dot plot, calculate the mean and MAD, describe the shape of the distribution, and answer questions.

**SAMPLE PROBLEM**  
*(From Lesson 11)*

Insects chirp more as the air gets warmer. You want to answer the following question: Are field crickets better predictors of air temperature than katydids? The following data are the number of chirps per minute for 10 insects of each type. All the data were taken on the same evening at the same time.

<table>
<thead>
<tr>
<th>Insect</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Cricket Chirps</td>
<td>35</td>
<td>32</td>
<td>35</td>
<td>37</td>
<td>34</td>
<td>34</td>
<td>38</td>
<td>35</td>
<td>36</td>
<td>34</td>
</tr>
<tr>
<td>Number of Katydid Chirps</td>
<td>66</td>
<td>62</td>
<td>61</td>
<td>64</td>
<td>63</td>
<td>62</td>
<td>68</td>
<td>64</td>
<td>66</td>
<td>64</td>
</tr>
</tbody>
</table>

a. Draw a dot plot for each data set by using the same scale, ranging from 30 to 70. Visually, what conclusions can you draw from the data plots?

Visually, you can see that the value for the mean number of chirps is higher for the katydids than for the crickets. The variability appears to be similar.

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**TERMS**

**Absolute deviation:** The distance of a data value from the mean of the data set.

**Mean:** The average of the values in a data set, also interpreted as a fair share (equal share of the total) or a balance point (where the sums of the distances, or absolute deviations, above and below the mean are equal). Mean is used as a measure of center in an approximately symmetric data distribution.

**Mean absolute deviation (MAD):** In a numerical data set, the MAD is the average of the distance (absolute deviation) each value is from the mean of the data set. It is found by determining the mean of the entire data set and the distance of each individual value from that mean and then calculating the mean of those distances. (See Sample Problems.)

**HOW YOU CAN HELP AT HOME**

You can help at home in many ways. Here are some tips to help you get started.

- With your child, discuss the point where the data balance on the dot plot above. (Answer: The balance point from the dot plot above must be 4 in order for the total of the distances on either side of the mean to be equal. At point 4, the sum of the absolute deviations to the right is 4 since 8 is 4 units away from 4, and the sum of the absolute deviations to the left is 4 since 3 is 1 unit away, 1 is 3 units away, and 1 + 3 = 4.)

- The number of red cubes in six bags of cubes is as follows: 4, 4, 4, 4, 4, 4. Find the mean and MAD. (Mean: 4, MAD: 0. All data values are the same so there is no variability.) Various colored cubes are mixed up and the bags are filled again. Now, the number of red cubes in each bag is 1, 2, 4, 5, 6, 6. What happens to the mean? (The mean does not change.) Without doing any calculations, explain how the MAD is affected when the cubes are mixed up and the bags are refilled. Is the new MAD the same or larger? (The MAD is larger because now there is variability, so the MAD is greater than zero.)

**SAMPLE PROBLEM**

b. Calculate the mean and MAD for each distribution.

*Crickets:* The mean is 35 chirps per minute because the sum of the number of chirps is 350 and $350 \div 10 = 35$.

The sum of all the deviations (distances from the mean) is 12 because

$0 + 3 + 0 + 2 + 1 + 1 + 3 + 0 + 1 + 1 = 12$. The MAD is \( \frac{12}{10} = 1.2 \) chirps per minute because

*Katydid:* The mean is 64 chirps per minute because the sum of the number of chirps is 640 and $640 \div 10 = 64$.

The sum of all the deviations is 16 because $2 + 2 + 3 + 0 + 1 + 2 + 4 + 0 + 2 + 0 = 16$. The MAD is \( \frac{16}{10} = 1.6 \) chirps per minute because

Conclusion: Based on the data from the full problem in Lesson 11, field crickets are not better predictors of air temperature than katydids.

Additional sample problems with detailed answer steps are found in the Eureka Math Homework Helpers books. Learn more at GreatMinds.org.
This topic focuses on data distributions (data sets) that are skewed (i.e., the tail is toward the smaller or larger value). Students will use the median as a measure of center and the interquartile range (IQR) as the measure of variability to describe these data distributions. Students calculate the values in the five-number summary (minimum, lower quartile, median, upper quartile, and maximum) of a data distribution in order to make a box plot and to interpret and compare data distributions.

You can expect to see homework that asks your child to do the following:

- Use data presented in a table or a dot plot to calculate the mean, median, and IQR of a data set.
- Create a data set with a specified number of values that satisfies certain characteristics, such as where the IQR is equal to the range (maximum minus minimum).
- Given a set of data, analyze the effect that adding certain values to the data set would have on the IQR.
- Analyze the differences between data represented in a dot plot and other data represented in a box plot.
- Given a dot plot or a set of data, determine the values in the five-number summary and use those values to create a box plot.
- Describe the center and spread (variability) of a box plot and use a box plot to answer real-world questions.

### SAMPLE PROBLEM  (From Lesson 15)

The maximum speeds of selected birds are given in the table.

<table>
<thead>
<tr>
<th>Bird</th>
<th>Speed (mph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peregrine falcon</td>
<td>242</td>
</tr>
<tr>
<td>Swift</td>
<td>120</td>
</tr>
<tr>
<td>Spine-tailed swift</td>
<td>106</td>
</tr>
<tr>
<td>White-throated needle tail</td>
<td>105</td>
</tr>
<tr>
<td>Eurasian hobby</td>
<td>100</td>
</tr>
<tr>
<td>Pigeon</td>
<td>100</td>
</tr>
<tr>
<td>Frigatebird</td>
<td>95</td>
</tr>
<tr>
<td>Spur-winged goose</td>
<td>88</td>
</tr>
<tr>
<td>Red-breasted merganser</td>
<td>80</td>
</tr>
<tr>
<td>Canvasback duck</td>
<td>72</td>
</tr>
<tr>
<td>Anna’s hummingbird</td>
<td>61.06</td>
</tr>
<tr>
<td>Ostrich</td>
<td>60</td>
</tr>
</tbody>
</table>

a. Describe the variability in the birds’ speeds. Explain your reasoning.

*It looks like the maximum speeds of the birds vary significantly since they go from 60 mph to 242 mph.*

b. Determine the five-number summary for the speeds in the data set. What does the five-number summary tell you about the distribution of speeds for the data set?

*Five-number summary: Min = 60, Q1 = 76, Median = 97.5, Q3 = 105.5, Max = 242*

*The summary gives me a sense of the range, or span, of the speeds (maximum speed minus minimum speed) and how the speeds are grouped around the median.*
SAMPLE PROBLEM (continued)

c. Use the five-number summary to make a box plot for the data set.

\[ \text{Maximum Speed of Selected Birds} \]

\[ \text{Speed of Birds (mph)} \]

\[ 0 \quad 25 \quad 50 \quad 75 \quad 100 \quad 125 \quad 150 \quad 175 \quad 200 \quad 225 \quad 250 \]

\[ \text{d. Write several sentences describing the speeds of the birds.} \]

Answers will vary. One bird listed in this table has a very high maximum speed (i.e., the falcon, at 242 mph). Three-fourths of the birds fly slower than 106 mph, and the slowest bird flies 60 mph.

Additional sample problems with detailed answer steps are found in the Eureka Math Homework Helpers books. Learn more at GreatMinds.org.

HOW YOU CAN HELP AT HOME

You can help at home in many ways. Here are some tips to help you get started.

▪ Challenge your child to create a data set with seven or more values, in which the minimum is 5, the range is 20, and the median is 16 (e.g., 5, 7, 10, 16, 19, 22, 25 or 5, 5, 15, 16, 21, 24, 25).

▪ Invite your child to create a data set with at least 10 values and then determine the five-number summary. (See Sample Problem.) Have your child sketch a box plot to represent the data and describe the variability and center of the data. Ask, “What information does the interquartile range give you about the data?” (It tells you how spread out the data are.)

TERMS

Interquartile range (IQR): A measure of variability for skewed data distributions that describes how spread out the middle 50 percent of the data are. The IQR is calculated by subtracting the lower quartile (Q1) from the upper quartile (Q3) of a data set (i.e., IQR = Q3 − Q1).

Lower quartile: The median of the bottom half of the values in a data set.

Median: A measure of center in a skewed data distribution. If the data set has an odd number of values, the median is the middle number after ordering the values from least to greatest. If the data set has an even number of values, the median is halfway between the two middle values. For example, if the two middle values in a data set are 7 and 11, the median is 9.

Upper quartile: The median of the top half of the values in a data set.

MODELS

Box Plot

For more resources, visit Eureka.support

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KEY CONCEPT OVERVIEW

In this topic, students develop a statistical project. They use the four-step method: write a statistical question, collect appropriate data, construct appropriate graphical and numerical summaries, and write an answer to the statistical question. Students revisit the graphs (dot plots, histograms, and box plots) and numerical summaries (mean, mean absolute deviation [MAD], median, and interquartile range) that they have worked with throughout the module. They choose the best representations to integrate in their projects, answering real-world questions and furthering their understanding of shape, center, and variability in a data set.

You can expect to see homework that asks your child to do the following:

- Develop or choose a statistical question from previous lessons. Create and explain a plan for collecting and summarizing data. Complete the statistical project.
- Match a histogram to the appropriate set of summary measures (minimum, lower quartile, median, upper quartile, maximum, mean, and MAD).
- Given a dot plot, determine the five-number summary and then create a histogram.
- Match the data presented in a histogram to the dot plot that represents the data.
- Answer questions by analyzing the data shown in a dot plot, box plot, and/or histogram.

SAMPLE PROBLEMS  (Lesson 18)

Here is a data set of the ages (in years) of 43 participants who ran in a 5-kilometer race.

<table>
<thead>
<tr>
<th>Age</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>30</td>
<td>3</td>
</tr>
<tr>
<td>35</td>
<td>1</td>
</tr>
<tr>
<td>36</td>
<td>1</td>
</tr>
<tr>
<td>34</td>
<td>1</td>
</tr>
<tr>
<td>38</td>
<td>1</td>
</tr>
<tr>
<td>45</td>
<td>1</td>
</tr>
<tr>
<td>43</td>
<td>1</td>
</tr>
<tr>
<td>23</td>
<td>1</td>
</tr>
<tr>
<td>47</td>
<td>1</td>
</tr>
<tr>
<td>27</td>
<td>1</td>
</tr>
<tr>
<td>21</td>
<td>1</td>
</tr>
<tr>
<td>30</td>
<td>1</td>
</tr>
<tr>
<td>32</td>
<td>2</td>
</tr>
<tr>
<td>31</td>
<td>1</td>
</tr>
<tr>
<td>32</td>
<td>1</td>
</tr>
<tr>
<td>36</td>
<td>1</td>
</tr>
<tr>
<td>74</td>
<td>1</td>
</tr>
<tr>
<td>41</td>
<td>1</td>
</tr>
<tr>
<td>41</td>
<td>1</td>
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<tr>
<td>51</td>
<td>1</td>
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<tr>
<td>61</td>
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<tr>
<td>50</td>
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<td>49</td>
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<td>37</td>
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</tr>
<tr>
<td>41</td>
<td>1</td>
</tr>
<tr>
<td>38</td>
<td>1</td>
</tr>
<tr>
<td>30</td>
<td>1</td>
</tr>
<tr>
<td>30</td>
<td>1</td>
</tr>
</tbody>
</table>

Here are summary statistics, a dot plot, and a histogram for the data:

Minimum = 18, Q1 = 30, Median = 35, Q3 = 41, Maximum = 74, Mean = 36.8, MAD = 8.1

For more resources, visit » Eureka.support
SAMPLE PROBLEMS  (continued)

a. For the dot plot and histogram, would you describe the shape of the data distribution as approximately symmetric or as skewed?

*Both graphs show a slightly skewed right data distribution.*

b. What is something you can see in the dot plot that is not as easy to see in the histogram?

*In the histogram, we cannot see exact values because the data are grouped in intervals, so we cannot determine the exact minimum or maximum age or the median age. Because the dot plot provides a dot for each observation, we can see the exact data values. We know the minimum is 18, the median is 35 (the middle value, or the 22nd observation, of the 43 observations), and the maximum is 74.*

c. Do the dot plot and the histogram seem to be centered in about the same place?

*Yes. Since both graphs are based on the same data, they should generally communicate the same information regarding the center.*

d. Do both the dot plot and the histogram convey information about the variability in the age distribution?

*Yes. Both graphs are based on the same data, so they generally communicate the same information about variability. However, in the dot plot, it is easy to see that the oldest runner (74) is an extreme departure from the other data. This is not as apparent in the histogram.*

Additional sample problems with detailed answer steps are found in the *Eureka Math Homework Helpers* books. Learn more at GreatMinds.org.

**HOW YOU CAN HELP AT HOME**

You can help at home in many ways. Here are some tips to help you get started.

- This module culminates in a four-step statistical project. Step one: Have your child explain the statistical question she wrote. Step two: How does she plan to collect the appropriate data? Step three: What graph (dot plot, histogram, or box plot) would work best to represent the data? What measures of center (mean or median) and variability (MAD or interquartile range) best represent the data? Step four: Have your child answer her statistical question.

- Your child will be required to present his completed statistical project. Throughout this topic, refer back to your child's project to monitor his progress. Encourage him to describe his work and, if applicable, to rehearse his presentation so he's more comfortable and confident when he presents the project.